INTEREST TABLES AND FORMULAE

JOHN C. HOLDEN

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WITH EXAMPLES AND DERIVATIONS

BY

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PREFACE

THE tables and formulæ contained in this work are intended primarily for the use of financial institutions, more especially those paying interest on deposits and loaning money on real estate.

The tables bearing on compound interest are designed for the convenience of the Bank and for ready reference in meeting the inquiries of its depositors and customers. They are computed from derived formulæ contained herein, with the aid of a ten-place logarithmic table.

The leap-year interest table is given as an economic feature, and its function as a saver, especially to Banks with large deposits, is obvious.

The limits of the tables, both as to rate and time, are regulated by the judgment of experience and though cases arising beyond these limits may present themselves for solution, they can readily be computed as shown by the examples under derivation of formulæ.

The computations of area are designed to meet the subdivisions of realty most commonly in use and are extended to cover cases which are possibilities though rare.

They can be used to verify as well as to compute.

The accompanying examples show the practical application of the tables and formulæ and demonstrate the manner of handling parallel cases.

JOHN G. HOLDEN

WASHINGTON, D. C.
1916

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INTEREST TABLES AND FORMULAE

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365 DAY BASIS

INTEREST

COMPARATIVE EXAMPLES

SEE TABLES 4, 5, AND 6
366 DAY BASIS

At 2%, 2½% and 3% per annum for 1 day

The interest for one day on the aggregate amount of daily balances is equal to the interest on the average balance for the given time and by the use of these tables (Nos. 1 to 6 incl.) more conveniently found.

365 DAY BASIS — **EXAMPLES** — 366 DAY BASIS

What is the interest on $2,304,800 at 2% per annum for one day?

TABLE NO. 1:		TABLE NO. 4:
109.589	2,000,000	109.290
16.438	300,000	16.394
.219	4,000	.219
.044	800	.044
$126.290	$2,304,800	$125.947

What is the interest on $1,890,300 at 2½% per annum for one day?

TABLE NO. 2:		TABLE NO. 5:
68.493	1,000,000	68.306
54.795	800,000	54.645
6.164	90,000	6.148
.021	300	.020
$129.473	$1,890,300	$129.119

What is the interest on $3,520,900 at 3% per annum for one day?

TABLE NO. 3:		TABLE NO. 6:
246.575	3,000,000	245.902
41.096	500,000	40.984
1.644	20,000	1.639
.074	900	.074
$289.389	$3,520,900	$288.599

DAILY BALANCE INTEREST TABLE AT 2% RATE, 365 DAY BASIS

TABLE NO. 1 — SHOWING INTEREST FOR ONE DAY — TABLE NO. 1

MILLIONS				THOUSANDS				HUNDREDS	
Principal	Interest	Principal	Interest	Principal	Interest	Principal	Interest	Prin.	Int.
1,000,000	54.795	21,000,000	1150.685	100,000	5.479	1,000	.055	100	.005
2,000,000	109.589	22,000,000	1205.479	200,000	10.959	2,000	.110	200	.011
3,000,000	164.384	23,000,000	1260.274	300,000	16.438	3,000	.164	300	.016
4,000,000	219.178	24,000,000	1315.068	400,000	21.918	4,000	.219	400	.022
5,000,000	273.973	25,000,000	1369.863	500,000	27.397	5,000	.274	500	.027
6,000,000	328.767	26,000,000	1424.658	600,000	32.877	6,000	.329	600	.033
7,000,000	383.562	27,000,000	1479.452	700,000	38.356	7,000	.384	700	.038
8,000,000	438.356	28,000,000	1534.247	800,000	43.836	8,000	.438	800	.044
9,000,000	493.151	29,000,000	1589.041	900,000	49.315	9,000	.493	900	.049
10,000,000	547.945	30,000,000	1643.826	.....	.....	10,000	.548		
11,000,000	602.740	31,000,000	1698.620	.....	.....	20,000	1.096		
12,000,000	657.534	32,000,000	1753.425	.....	.....	30,000	1.644		
13,000,000	712.329	33,000,000	1808.219	.....	.....	40,000	2.192		
14,000,000	767.123	34,000,000	1863.014	.....	.....	50,000	2.740		
15,000,000	821.918	35,000,000	1917.808	.....	.....	60,000	3.288		
16,000,000	876.712	36,000,000	1972.603	.....	.....	70,000	3.836		
17,000,000	931.507	37,000,000	2027.397	.....	.....	80,000	4.383		
18,000,000	986.301	38,000,000	2082.192	.....	.....	90,000	4.931		
19,000,000	1041.096	39,000,000	2136.986						
20,000,000	1095.890	40,000,000	2191.781						

DAILY BALANCE INTEREST TABLE AT 2½% RATE, 365 DAY BASIS

TABLE NO. 2 — SHOWING INTEREST FOR ONE DAY — TABLE NO. 2

MILLIONS				THOUSANDS				HUNDREDS	
Principal	Interest	Principal	Interest	Principal	Interest	Principal	Interest	Prin.	Int.
1,000,000	68.493	21,000,000	1438.356	100,000	6.849	1,000	.068	100	.007
2,000,000	136.986	22,000,000	1506.849	200,000	13.699	2,000	.137	200	.014
3,000,000	205.479	23,000,000	1575.342	300,000	20.548	3,000	.205	300	.021
4,000,000	273.973	24,000,000	1643.836	400,000	27.397	4,000	.274	400	.027
5,000,000	342.466	25,000,000	1712.329	500,000	34.247	5,000	.342	500	.034
6,000,000	410.959	26,000,000	1780.822	600,000	41.096	6,000	.411	600	.041
7,000,000	479.452	27,000,000	1849.315	700,000	47.945	7,000	.479	700	.048
8,000,000	547.945	28,000,000	1917.808	800,000	54.795	8,000	.548	800	.055
9,000,000	616.438	29,000,000	1986.301	900,000	61.644	9,000	.616	900	.062
10,000,000	684.932	30,000,000	2054.795	.....	.....	10,000	.685		
11,000,000	753.425	31,000,000	2123.288	.....	.....	20,000	1.370		
12,000,000	821.918	32,000,000	2191.781	.....	.....	30,000	2.055		
13,000,000	890.411	33,000,000	2260.274	.....	.....	40,000	2.740		
14,000,000	958.904	34,000,000	2328.767	.....	.....	50,000	3.425		
15,000,000	1027.397	35,000,000	2397.260	.....	.....	60,000	4.110		
16,000,000	1095.890	36,000,000	2465.753	.....	.....	70,000	4.795		
17,000,000	1164.384	37,000,000	2534.247	.....	.....	80,000	5.479		
18,000,000	1232.877	38,000,000	2602.740	.....	.....	90,000	6.164		
19,000,000	1301.370	39,000,000	2671.233	.....					
20,000,000	1369.863	40,000,000	2739.726						

365 DAY BASIS

2½%

DAILY BALANCE INTEREST TABLE AT 3% RATE, 365 DAY BASIS

TABLE No. 3 — SHOWING INTEREST FOR ONE DAY — TABLE No. 3

MILLIONS				THOUSANDS				HUNDREDS	
Principal	Interest	Principal	Interest	Principal	Interest	Principal	Interest	Prin.	Int.
1,000,000	82.192	21,000,000	1726.027	100,000	8.219	1,000	.082	100	.008
2,000,000	164.384	22,000,000	1808.219	200,000	16.438	2,000	.164	200	.016
3,000,000	246.575	23,000,000	1890.411	300,000	24.658	3,000	.247	300	.025
4,000,000	328.767	24,000,000	1972.603	400,000	32.877	4,000	.329	400	.033
5,000,000	410.959	25,000,000	2054.795	500,000	41.096	5,000	.411	500	.041
6,000,000	493.151	26,000,000	2136.986	600,000	49.315	6,000	.493	600	.049
7,000,000	575.342	27,000,000	2219.178	700,000	57.534	7,000	.575	700	.058
8,000,000	657.534	28,000,000	2301.370	800,000	65.753	8,000	.658	800	.066
9,000,000	739.726	29,000,000	2383.562	900,000	73.973	9,000	.740	900	.074
10,000,000	821.918	30,000,000	2465.753	.....	.....	10,000	.822		
11,000,000	904.110	31,000,000	2547.945	.....	.....	20,000	1.644		
12,000,000	986.301	32,000,000	2630.137	.....	.....	30,000	2.466		
13,000,000	1068.493	33,000,000	2712.329	.....	.....	40,000	3.288		
14,000,000	1150.685	34,000,000	2794.521	.....	.....	50,000	4.110		
15,000,000	1232.877	35,000,000	2876.712	.....	.....	60,000	4.932		
16,000,000	1315.069	36,000,000	2958.904	.....	.....	70,000	5.753		
17,000,000	1397.261	37,000,000	3041.096	.....	.....	80,000	6.575		
18,000,000	1479.452	38,000,000	3123.288	.....	.....	90,000	7.397		
19,000,000	1561.644	39,000,000	3205.479						
20,000,000	1643.836	40,000,000	3287.671						

365 DAY BASIS

3%

LEAP YEAR INTEREST TABLES

366 DAY BASIS

See Tables 4, 5, and 6

The use of these tables during leap year means an annual saving to the Bank on every million dollars ($1,000,000) of deposits:

At 2 %—$54.90

2½%—$68.62

3 %—$82.35

Higher rates in proportion.

LEAP YEAR—366 DAY BASIS

DAILY BALANCE INTEREST TABLE

TABLE NO. 4 AT 2% RATE, SHOWING INTEREST FOR ONE DAY TABLE NO. 4

MILLIONS				THOUSANDS				HUNDREDS	
Principal	Interest	Principal	Interest	Principal	Interest	Principal	Interest	Prin.	Int.
1,000,000	54.645	21,000,000	1147.541	100,000	5.465	1,000	.055	100	.005
2,000,000	109.290	22,000,000	1202.186	200,000	10.929	2,000	.110	200	.011
3,000,000	163.934	23,000,000	1256.831	300,000	16.394	3,000	.164	300	.016
4,000,000	218.579	24,000,000	1311.475	400,000	21.858	4,000	.219	400	.022
5,000,000	273.224	25,000,000	1366.120	500,000	27.323	5,000	.273	500	.027
6,000,000	327.869	26,000,000	1420.765	600,000	32.787	6,000	.328	600	.033
7,000,000	382.514	27,000,000	1475.410	700,000	38.252	7,000	.383	700	.038
8,000,000	437.159	28,000,000	1530.055	800,000	43.716	8,000	.437	800	.044
9,000,000	491.803	29,000,000	1584.699	900,000	49.181	9,000	.492	900	.049
10,000,000	546.448	30,000,000	1639.344	.....	.....	10,000	.547		
11,000,000	601.093	31,000,000	1693.989	.....	.....	20,000	1.093		
12,000,000	655.738	32,000,000	1748.634	.....	.....	30,000	1.640		
13,000,000	710.383	33,000,000	1803.279	.....	.....	40,000	2.186		
14,000,000	765.027	34,000,000	1857.923	.....	.....	50,000	2.732		
15,000,000	819.672	35,000,000	1912.568	.....	.....	60,000	3.279		
16,000,000	874.317	36,000,000	1967.213	.....	.....	70,000	3.825		
17,000,000	928.962	37,000,000	2021.858	.....	.....	80,000	4.372		
18,000,000	983.607	38,000,000	2076.503	.....	.....	90,000	4.918		
19,000,000	1038.251	39,000,000	2131.148						
20,000,000	1092.896	40,000,000	2185.792						

LEAP YEAR—366 DAY BASIS

DAILY BALANCE INTEREST TABLE

TABLE NO. 5 AT 2½% RATE, SHOWING INTEREST FOR ONE DAY TABLE NO. 5

MILLIONS				THOUSANDS				HUNDREDS	
Principal	Interest	Principal	Interest	Principal	Interest	Principal	Interest	Prin.	Int.
1,000,000	68.306	21,000,000	1434.426	100,000	6.831	1,000	.068	100	.007
2,000,000	136.612	22,000,000	1502.732	200,000	13.661	2,000	.137	200	.014
3,000,000	204.918	23,000,000	1571.038	300,000	20.492	3,000	.205	300	.020
4,000,000	273.224	24,000,000	1639.344	400,000	27.322	4,000	.273	400	.027
5,000,000	341.530	25,000,000	1707.650	500,000	34.153	5,000	.342	500	.034
6,000,000	409.836	26,000,000	1775.956	600,000	40.984	6,000	.410	600	.041
7,000,000	478.142	27,000,000	1844.262	700,000	47.814	7,000	.478	700	.048
8,000,000	546.448	28,000,000	1912.568	800,000	54.645	8,000	.546	800	.055
9,000,000	614.754	29,000,000	1980.874	900,000	61.475	9,000	.615	900	.061
10,000,000	683.060	30,000,000	2049.180	.....	.....	10,000	.683		
11,000,000	751.366	31,000,000	2117.486	.....	.....	20,000	1.366		
12,000,000	819.672	32,000,000	2185.792	.....	.....	30,000	2.049		
13,000,000	887.978	33,000,000	2254.098	.....	.....	40,000	2.732		
14,000,000	956.284	34,000,000	2322.404	.....	.....	50,000	3.415		
15,000,000	1024.590	35,000,000	2390.710	.....	.....	60,000	4.098		
16,000,000	1092.896	36,000,000	2459.016	.....	.....	70,000	4.781		
17,000,000	1161.202	37,000,000	2527.322	.....	.....	80,000	5.464		
18,000,000	1229.508	38,000,000	2595.628	.....	.....	90,000	6.148		
19,000,000	1297.814	39,000,000	2663.934						
20,000,000	1366.120	40,000,000	2732.240						

Leap Year—366 Day Basis

DAILY BALANCE INTEREST TABLE

Table No. 6 — At 3% Rate, Showing Interest for One Day — Table No. 6

Millions				Thousands				Hundreds	
Principal	Interest	Principal	Interest	Principal	Interest	Principal	Interest	Prin.	Int.
1,000,000	81.967	21,000,000	1721.311	100,000	8.197	1,000	.082	100	.008
2,000,000	163.934	22,000,000	1803.279	200,000	16.393	2,000	.164	200	.016
3,000,000	245.902	23,000,000	1885.246	300,000	24.590	3,000	.246	300	.025
4,000,000	327.869	24,000,000	1967.213	400,000	32.787	4,000	.328	400	.033
5,000,000	409.836	25,000,000	2049.180	500,000	40.984	5,000	.410	500	.041
6,000,000	491.803	26,000,000	2131.148	600,000	49.180	6,000	.492	600	.049
7,000,000	573.770	27,000,000	2213.115	700,000	57.377	7,000	.574	700	.057
8,000,000	655.738	28,000,000	2295.082	800,000	65.574	8,000	.656	800	.066
9,000,000	737.705	29,000,000	2377.049	900,000	73.771	9,000	.738	900	.074
10,000,000	819.672	30,000,000	2459.016	.....	.....	10,000	.820		
11,000,000	901.639	31,000,000	2540.984	.....	.....	20,000	1.639		
12,000,000	983.607	32,000,000	2622.951	.....	.....	30,000	2.459		
13,000,000	1065.574	33,000,000	2704.918	.....	.....	40,000	3.279		
14,000,000	1147.541	34,000,000	2786.885	.....	.....	50,000	4.098		
15,000,000	1229.508	35,000,000	2868.852	.....	.....	60,000	4.918		
16,000,000	1311.475	36,000,000	2950.820	.....	.....	70,000	5.738		
17,000,000	1393.443	37,000,000	3032.787	.....	.....	80,000	6.557		
18,000,000	1475.410	38,000,000	3114.754	.....	.....	90,000	7.377		
19,000,000	1557.377	39,000,000	3196.721						
20,000,000	1639.344	40,000,000	3278.689						

2%

To find what amount any sum will produce at 2% per annum, interest compounded at tabular intervals.*

See Table No. 7. Page 11

Computed by Formula No. II Page 121

EXAMPLE

What will $450.00 amount to in 6 years at 2% per annum, interest compounded quarterly?

Tabular amount †	=	1.1271597	
Sum	=	450.	
Amount	=	$507.22	Answer.

* Tabular intervals: Monthly, Bi-monthly, Quarterly, Semi-annually, Annually.

† For tabular amount, see Table No. 7, vertically under quarterly, horizontally from 6 years.

THE AMOUNT OF $1 AT 2% INTEREST COMPOUNDED AT TABULAR INTERVALS

TABLE No. 7 $A = (1 + r)^m$ TABLE No. 7

	Monthly	Bi-monthly	Quarterly	Semi-annually	Annually
1 year.....	1.0201843	1.0201674	1.0201505	1.0201000	1.0200000
2 years.....	1.0407760	1.0407415	1.0407070	1.0406040	1.0404000
3 "	1.0617834	1.0617305	1.0616778	1.0615201	1.0612080
4 "	1.0832148	1.0831429	1.0830711	1.0828567	1.0824321
5 "	1.1050788	1.1049871	1.1048955	1.1046221	1.1040808
6 "	1.1273841	1.1272718	1.1271597	1.1268250	1.1261624
7 "	1.1501396	1.1500059	1.1498726	1.1494742	1.1486856
8 "	1.1733544	1.1731986	1.1730431	1.1725786	1.1716593
9 "	1.1970378	1.1968589	1.1966805	1.1961474	1.1950925
10 "	1.2211992	1.2209965	1.2207942	1.2201900	1.2189944

$2\frac{1}{2}\%$

To find what amount any sum will produce at $2\frac{1}{2}\%$ per annum, interest compounded at tabular intervals.*

See Table No. 8
Page 13

Computed by Formula No. II
Page 121

EXAMPLE

What will $1047.00 amount to in 4 years at $2\frac{1}{2}\%$ per annum, interest compounded semi-annually?

Tabular amount †	=	1.1044861	
Sum	=	1047.	
Amount	=	$1156.39	Answer.

* Tabular intervals: Monthly, Bi-monthly, Quarterly, Semi-annually, Annually.

† For tabular amount, see Table No. 8, vertically under semi-annually, horizontally from 4 years.

THE AMOUNT OF $1 AT 2½%, INTEREST COMPOUNDED AT TABULAR INTERVALS

TABLE No. 8 TABLE No. 8

	Monthly	Bi-monthly	Quarterly	Semi-annually	Annually
1 year.....	1.0252884	1.0252618	1.0252353	1.0251562	1.0250000
2 years.....	1.0512164	1.0511618	1.0511075	1.0509453	1.0506250
3 "	1.0778000	1.0777161	1.0776325	1.0773831	1.0768906
4 "	1.1050559	1.1049413	1.1048270	1.1044861	1.1038128
5 "	1.1330010	1.1328541	1.1327077	1.1322708	1.1314082
6 "	1.1616529	1.1614721	1.1612919	1.1607545	1.1596934
7 "	1.1910293	1.1908131	1.1905975	1.1899547	1.1886857
8 "	1.2211485	1.2208952	1.2206427	1.2198895	1.2184028
9 "	1.2520295	1.2517373	1.2514460	1.2505773	1.2488629
10 "	1.2836914	1.2833585	1.2830267	1.2820372	1.2800845

3%

To find what amount any sum will produce at 3% per annum, interest compounded at tabular intervals.*

See Table No. 9
Page 15

Computed by Formula No. II
Page 121

EXAMPLE

What will $95,764.00 amount to in 5 years at 3% per annum interest, compounded bi-monthly?

Tabular amount †	=	1.1614000	
Sum	=	95,764.	
Amount	=	$111,220.30	Answer.

* Tabular intervals: Monthly, Bi-monthly, Quarterly, Semi-annually, Annually.

† For tabular amount, see Table No. 9, vertically under bi-monthly, horizontally from 5 years.

THE AMOUNT OF $1 AT 3%, INTEREST COMPOUNDED AT TABULAR INTERVALS

TABLE No. 9

TABLE No. 9

	Monthly	Bi-monthly	Quarterly	Semi-annually	Annually
1 year.....	1.0304159	1.0303775	1.0303391	1.0302250	1.0300000
2 years.....	1.0617570	1.0616778	1.0615988	1.0613635	1.0609000
3 "	1.0940514	1.0939289	1.0938068	1.0934432	1.0927270
4 "	1.1273280	1.1271597	1.1269921	1.1264925	1.1255088
5 "	1.1616167	1.1614000	1.1611841	1.1605408	1.1592740
6 "	1.1969484	1.1966805	1.1964135	1.1956181	1.1940522
7 "	1.2333548	1.2330327	1.2327117	1.2317557	1.2298738
8 "	1.2708684	1.2704891	1.2701112	1.2689855	1.2667700
9 "	1.3095231	1.3090834	1.3086453	1.3073406	1.3047731
10 "	1.3493535	1.3488501	1.3483486	1.3468550	1.3439163

$3\frac{1}{2}\%$

To find what amount any sum will produce at $3\frac{1}{2}\%$ per annum, interest compounded at tabular intervals.*

See Table No. 10
Page 17

Computed by Formula No. II
Page 121

EXAMPLE

What will $642.87 amount to in 7 years at $3\frac{1}{2}\%$ per annum, interest compounded monthly?

Tabular amount †	=	1.2771657	
Sum	=	642.87	
Amount	=	$821.05	Answer.

* Tabular intervals: Monthly, Bi-monthly, Quarterly, Semi-annually, Annually.

† For tabular amount see Table No. 10, vertically under monthly, horizontally from 7 years.

THE AMOUNT OF $1 AT 3½%, INTEREST COMPOUNDED AT TABULAR INTERVALS

TABLE No. 10 TABLE No. 10

	Monthly	Bi-monthly	Quarterly	Semi-annually	Annually
1 year.....	1.0355669	1.0355143	1.0354620	1.0353062	1.0350000
2 years.....	1.0723988	1.0722900	1.0721816	1.0718590	1.0712250
3 "	1.1105408	1.1103718	1.1102034	1.1097023	1.1087178
4 "	1.1500393	1.1498059	1.1495735	1.1488817	1.1475230
5 "	1.1909427	1.1906406	1.1903397	1.1894444	1.1876863
6 "	1.2333009	1.2329255	1.2325516	1.2314393	1.2292553
7 "	1.2771657	1.2767121	1.2762604	1.2749168	1.2722792
8 "	1.3225906	1.3220538	1.3215193	1.3199293	1.3168090
9 "	1.3696311	1.3690057	1.3683830	1.3665311	1.3628973
10 "	1.4183447	1.4176251	1.4169087	1.4147781	1.4105987

4%

To find what amount any sum will produce at 4% per annum, interest compounded at tabular intervals.*

See Table No. 11 [Page 19

Computed by Formula No. II Page 121

EXAMPLE

What will $10,140.97 amount to in 9 years at 4% per annum, interest compounded annually?

Tabular amount †	=	1.4233118	
Sum	=	10,140.97	
Amount	=	$14,433.76	Answer.

* Tabular intervals: Monthly, Bi-monthly, Quarterly, Semi-annually, Annually.

† For tabular amount see Table No. 11, vertically under annually, horizontally from 9 years.

THE AMOUNT OF $1 AT 4%, INTEREST COMPOUNDED AT TABULAR INTERVALS

TABLE No. 11 TABLE No. 11

	Monthly	Bi-monthly	Quarterly	Semi-annually	Annually
1 year.....	1.0407415	1.0406726	1.0406040	1.0404000	1.0400000
2 years.....	1.0831429	1.0829994	1.0828567	1.0824321	1.0816000
3 "	1.1272718	1.1270479	1.1268250	1.1261624	1.1248640
4 "	1.1731986	1.1728879	1.1725786	1.1716593	1.1698585
5 "	1.2209965	1.2205923	1.2201900	1.2189944	1.2166529
6 "	1.2707417	1.2702370	1.2697346	1.2682418	1.2653190
7 "	1.3225137	1.3219008	1.3212909	1.3194787	1.3159317
8 "	1.3763949	1.3756660	1.3749406	1.3727857	1.3685690
9 "	1.4324714	1.4316179	1.4307687	1.4282462	1.4233118
10 "	1.4908325	1.4898456	1.4888637	1.4859474	1.4802442

$4\frac{1}{2}\%$

To find what amount any sum will produce at $4\frac{1}{2}\%$ per annum, interest compounded at tabular intervals.*

See Table No. 12
Page 21

Computed by Formula No. II
Page 121

EXAMPLE

What will $594.67 amount to in 10 years at $4\frac{1}{2}\%$ per annum, interest compounded quarterly?

Tabular amount †	=	1.5643767	
Sum	=	594.67	
Amount	=	$930.28	Answer.

* Tabular intervals: Monthly, Bi-monthly, Quarterly, Semi-annually, Annually.

† For tabular amount see Table No. 12, vertically under quarterly, horizontally from 10 years.

THE AMOUNT OF $1 AT 4½%, INTEREST COMPOUNDED AT TABULAR INTERVALS

TABLE No. 12 TABLE No. 12

	Monthly	Bi-monthly	Quarterly	Semi-annually	Annually
1 year.....	1.0459398	1.0458522	1.0457650	1.0455062	1.0450000
2 years...	1.0939900	1.0938068	1.0936246	1.0930833	1.0920250
3 " ...	1.1442477	1.1439603	1.1436744	1.1428254	1.1411661
4 " ...	1.1968143	1.1964135	1.1960147	1.1948311	1.1925186
5 " ...	1.2517957	1.2512717	1.2507504	1.2492034	1.2461819
6 "	1.3093029	1.3086453	1.3079911	1.3060499	1.3022601
7 "	1.3694521	1.3686496	1.3678515	1.3654834	1.3608618
8 "	1.4323644	1.4314053	1.4304513	1.4276214	1.4221006
9 "	1.4981670	1.4970384	1.4959160	1.4925871	1.4860951
10 "	1.5669925	1.5656810	1.5643767	1.5605092	1.5529694

5%

To find what amount any sum will produce at 5% per annum, interest compounded at tabular intervals.*

See Table No. 13
Page 23

Computed by Formula No. II
Page 121

EXAMPLE

What will $12,500.00 amount to in 6 years at 5% per annum, interest compounded semi-annually?

Tabular amount †	=	1.3448888	
Sum	=	12,500.	
Amount	=	$16,811.11	Answer.

* Tabular intervals: Monthly, Bi-monthly, Quarterly, Semi-annually, Annually.

† For tabular amount see Table No. 13, vertically under semi-annually, horizontally from 6 years.

THE AMOUNT OF $1 AT 5%, INTEREST COMPOUNDED AT TABULAR INTERVALS

TABLE No. 13 TABLE No. 13

	Monthly	Bi-monthly	Quarterly	Semi-annually	Annually
1 year.....	1.0511618	1.0510533	1.0509453	1.0506250	1.0500000
2 years.....	1.1049413	1.1047130	1.1044861	1.1038128	1.1025000
3 "	1.1614721	1.1611123	1.1607545	1.1596934	1.1576250
4 "	1.2208952	1.2203909	1.2198895	1.2184028	1.2155062
5 "	1.2833585	1.2826959	1.2820372	1.2800845	1.2762815
6 "	1.3490176	1.3481817	1.3473510	1.3448888	1.3400956
7 "	1.4180359	1.4170109	1.4159923	1.4129738	1.4071004
8 "	1.4905853	1.4893540	1.4881305	1.4845056	1.4774554
9 "	1.5668464	1.5653904	1.5639438	1.5596587	1.5513282
10 "	1.6470092	1.6453088	1.6436194	1.6386164	1.6288946

$5\frac{1}{2}\%$

To find what amount any sum will produce at $5\frac{1}{2}\%$ per annum, interest compounded at tabular intervals.*

See Table No. 14
Page 25

Computed by Formula No. II
Page 121

EXAMPLE

What will $3450.75 amount to in 7 years at $5\frac{1}{2}\%$ per annum, interest compounded bi-monthly?

Tabular amount †	=	1.4670389	
Sum	=	3450.75	
Amount	=	$5062.38	Answer.

* Tabular intervals: Monthly, Bi-monthly, Quarterly, Semi-annually, Annually.

† For tabular amount, see Table No. 14, vertically under bi-monthly, horizontally from 7 years.

THE AMOUNT OF $1 AT 5½%, INTEREST COMPOUNDED AT TABULAR INTERVALS

TABLE No. 14 TABLE No. 14

	Monthly	Bi-monthly	Quarterly	Semi-annually	Annually
1 year.....	1.0564078	1.0562759	1.0561448	1.0557562	1.0550000
2 years.....	1.1159975	1.1157188	1.1154418	1.1146212	1.1130250
3 "	1.1789485	1.1785069	1.1780681	1.1767683	1.1742413
4 "	1.2454505	1.2448284	1.2442105	1.2423805	1.2388246
5 "	1.3157037	1.3148823	1.3140664	1.3116510	1.3069600
6 "	1.3899197	1.3888785	1.3878444	1.3847837	1.3788428
7 "	1.4683221	1.4670389	1.4657647	1.4619941	1.4546791
8 "	1.5511470	1.5495979	1.5480598	1.5435094	1.5346865
9 "	1.6386438	1.6368030	1.6349753	1.6295697	1.6190942
10 "	1.7310762	1.7289156	1.7267706	1.7204284	1.7081444

6%

To find what amount any sum will produce at 6% per annum, interest compounded at tabular intervals.*

See Table No. 15
Page 27

Computed by Formula No. II
Page 121

EXAMPLE

What will $22,463.50 amount to in 4 years at 6% per annum, interest compounded quarterly?

Tabular amount †	=	1.2689855	
Sum	=	22,463.50	
Amount	=	$28,505.85	Answer.

* Tabular intervals: Monthly, Bi-monthly, Quarterly, Semi-annually, Annually.

† For tabular amount, see Table No. 15, vertically under quarterly, horizontally from 4 years.

THE AMOUNT OF $1 AT 6%, INTEREST COMPOUNDED AT TABULAR INTERVALS

TABLE No. 15 TABLE No. 15

	Monthly	Bi-monthly	Quarterly	Semi-annually	Annually
1 year.....	1.0616778	1.0615201	1.0613635	1.0609000	1.0600000
2 years.....	1.1271597	1.1268250	1.1264925	1.1255088	1.1236000
3 "	1.1966805	1.1961474	1.1956181	1.1940522	1.1910160
4 "	1.2704891	1.2697346	1.2689855	1.2667700	1.2624769
5 "	1.3488501	1.3478489	1.3468550	1.3439163	1.3382255
6 "	1.4320442	1.4307687	1.4295028	1.4257608	1.4185191
7 "	1.5203696	1.5187898	1.5172221	1.5125897	1.5036302
8 "	1.6141427	1.6122260	1.6103243	1.6047064	1.5938480
9 "	1.7136995	1.7114104	1.7091395	1.7024330	1.6894789
10 "	1.8193967	1.8166967	1.8140184	1.8061112	1.7908476

2%

To find what sum at 2% per annum, will produce any given amount, interest compounded at tabular intervals.*

See Table No. 16
Page 29

Computed by Formula No. III
Page 123

EXAMPLE

What sum will amount to $3000.00, interest compounded quarterly at 2% per annum for 7 years?

Tabular sum †	=	.8696616
Amount	=	3000.
Sum	=	$2608.99 Answer.

* Tabular intervals: Monthly, Bi-monthly, Quarterly, Semi-annually, Annually.

† For tabular sum, see Table No. 16, vertically under quarterly, horizontally from 7 years.

THE PRESENT VALUE OF $1, INTEREST COMPOUNDED AT TABULAR INTERVALS AT 2%

TABLE No. 16 TABLE No. 16

	Monthly	Bi-monthly	Quarterly	Semi-annually	Annually
1 year.....	.9802151	.9802313	.9802476	.9802961	.9803922
2 years.....	.9608216	.9608534	.9608853	.9609804	.9611688
3 "	.9418117	.9418586	.9419054	.9420453	.9423224
4 "	.9231780	.9232392	.9233005	.9234833	.9238455
5 "	.9049129	.9049880	.9050630	.9052870	.9057308
6 "	.8870092	.8870975	.8871858	.8874493	.8879714
7 "	.8694597	.8695608	.8696616	.8699630	.8705603
8 "	.8522575	.8523706	.8524836	.8528213	.8534905
9 "	.8353955	.8355204	.8356450	.8360174	.8367554
10 "	.8188673	.8190032	.8191389	.8195445	.8203484

$2\frac{1}{2}\%$

To find what sum at $2\frac{1}{2}\%$ per annum will produce any given amount, interest compounded at tabular intervals.*

See Table No. 17
Page 31

Computed by Formula No. III
Page 123

EXAMPLE

What sum will amount to $8350.00, interest compounded semi-annually at $2\frac{1}{2}\%$ per annum for 9 years?

Tabular sum †	=	.7996307	
Amount	=	8350.	
Sum	=	$6676.92	Answer.

* Tabular intervals: Monthly, Bi-monthly, Quarterly, Semi-annually, Annually.

† For tabular sum, see Table No. 17, vertically under semi-annually, horizontally from 9 years.

THE PRESENT VALUE OF $1, INTEREST COMPOUNDED AT TABULAR INTERVALS AT 2½%.

TABLE No. 17 — TABLE No. 17

	Monthly	Bi-monthly	Quarterly	Semi-annually	Annually
1 year.....	.9753353	.9753607	.9753859	.9754611	.9756098
2 years.....	.9512790	.9513284	.9513775	.9515243	.9518144
3 "	.9278160	.9278882	.9279602	.9281750	.9285995
4 "	.9049316	.9050255	.9051191	.9053985	.9059507
5 "	.8826118	.8827262	.8828403	.8831810	.8838543
6 "	.8608424	.8609764	.8611100	.8615086	.8622969
7 "	.8396100	.8397624	.8399144	.8403682	.8412653
8 "	.8189013	.8190712	.8192406	.8197464	.8207468
9 "	.7987033	.7988897	.7990757	.7996307	.8007284
10 "	.7790035	.7792055	.7794070	.7800086	.7811985

3%

To find what sum at 3% per annum, will produce any given amount, interest compounded at tabular intervals.*

See Table No. 18
Page 33

Computed by Formula No. III
Page 123

EXAMPLE

What sum will amount to $450.00, interest compounded monthly at 3% per annum for 4 years?

Tabular sum †	=	.8870533	
Amount	=	450.	
Sum	=	$399.18	Answer.

* Tabular intervals: Monthly, Bi-monthly, Quarterly, Semi-annually, Annually.

† For tabular sum, see Table No. 18, vertically under monthly, horizontally from 4 years.

THE PRESENT VALUE OF $1, INTEREST COMPOUNDED AT TABULAR INTERVALS AT 3%

TABLE No. 18 TABLE No. 18

	Monthly	Bi-monthly	Quarterly	Semi-annually	Annually
1 year.....	.9704819	.9705181	.9705543	.9706618	.9708738
2 years.....	.9418351	.9419054	.9419755	.9421843	.9425960
3 "	.9140339	.9141362	.9142383	.9145423	.9151417
4 "	.8870533	.8871858	.8873177	.8877113	.8884871
5 "	.8608692	.8610298	.8611899	.8616673	.8626089
6 "	.8354579	.8356450	.8358315	.8363875	.8374844
7 "	.8107967	.8110085	.8112197	.8118493	.8130916
8 "	.7868636	.7870985	.7873327	.7880311	.7894093
9 "	.7636368	.7638933	.7641490	.7649116	.7664168
10 "	.7410957	.7413723	.7416480	.7424705	.7440940

$3\frac{1}{2}\%$

To find what at $3\frac{1}{2}\%$ per annum, sum will produce any given amount, interest compounded at tabular intervals.*

See Table No. 19
Page 35

Computed by Formula No. III
Page 123

EXAMPLE

What sum will amount to $1400.00, interest compounded bi-monthly at $3\frac{1}{2}\%$ per annum for 5 years?

Tabular sum †	=	.8398840	
Amount	=	1400.	
Sum	=	$1175.84	Answer.

* Tabular intervals: Monthly, Bi-monthly, Quarterly, Semi-annually, Annually.

† For tabular sum, see Table No. 19, vertically under bi-monthly, horizontally from 5 years.

THE PRESENT VALUE OF $1, INTEREST COMPOUNDED AT TABULAR INTERVALS AT 3½%

TABLE No. 19 — TABLE No. 19

	Monthly	Bi-monthly	Quarterly	Semi-annually	Annually
1 year.....	.9656547	.9657037	.9657525	.9658978	.9661836
2 years.....	.9324890	.9325836	.9326779	.9329586	.9335107
3 "	.9004622	.9005993	.9007359	.9011426	.9019428
4 "	.8695355	.8697120	.8698879	.8704117	.8714423
5 "	.8396710	.8398840	.8400964	.8407287	.8419732
6 "	.8108322	.8110790	.8113251	.8120579	.8135007
7 "	.7829838	.7832620	.7835392	.7843649	.7859910
8 "	.7560919	.7563989	.7567048	.7576164	.7594116
9 "	.7301236	.7304572	.7307896	.7317800	.7337310
10 "	.7050473	.7054051	.7057618	.7068247	.7089189

4%

To find what sum at 4% per annum, will produce any given amount, interest compounded at tabular intervals.*

See Table No. 20
Page 37

Computed by Formula No. III
Page 123

EXAMPLE

What sum will amount to $892.50, interest compounded annually at 4% per annum for 6 years?

Tabular sum †	=	.7903146	
Amount	=	892.50	
Sum	=	$705.36	Answer.

* Tabular intervals: Monthly, Bi-monthly, Quarterly, Semi-annually, Annually.

† For tabular sum, see Table No. 20, vertically under annually, horizontally from 6 years.

THE PRESENT VALUE OF $1, INTEREST COMPOUNDED AT TABULAR INTERVALS AT 4%

TABLE No. 20 TABLE No. 20

	Monthly	Bi-monthly	Quarterly	Semi-annually	Annually
1 year.....	.9608534	.9609170	.9609804	.9611688	.9615385
2 years.....	.9232392	.9233615	.9234833	.9238455	.9245562
3 "	.8870975	.8872738	.8874493	.8879714	.8889964
4 "	.8523706	.8525964	.8528213	.8534905	.8548043
5 "	.8190032	.8192744	.8195445	.8203484	.8219271
6 "	.7869420	.7872547	.7875662	.7884932	.7903146
7 "	.7561359	.7564864	.7568356	.7578751	.7599179
8 "	.7265357	.7269207	.7273042	.7284458	.7306903
9 "	.6980942	.6985104	.6989250	.7001594	.7025868
10 "	.6707662	.6712105	.6716532	.6729714	.6755642

$4\frac{1}{2}\%$

To find what sum at $4\frac{1}{2}\%$ per annum, will produce any given amount, interest compounded at tabular intervals.*

See Table No. 21
Page 39

Computed by Formula No. III
Page 123

EXAMPLE

What sum will amount to $1960.00, interest compounded monthly at $4\frac{1}{2}\%$ per annum for 9 years?

Tabular sum †	=	.6674824	
Amount	=	1960.	
Sum	=	$1308.27	Answer.

* Tabular intervals: Monthly, Bi-monthly, Quarterly, Semi-annually, Annually.

† For tabular sum, see Table No. 21, vertically under monthly, horizontally from 9 years.

THE PRESENT VALUE OF $1, INTEREST COMPOUNDED AT TABLUAR INTERVALS AT 4½%

TABLE No. 21 TABLE No. 21

	Monthly	Bi-monthly	Quarterly	Semi-annually	Annually
1 year.....	.9560780	.9561581	.9562378	.9564745	.9569378
2 years.....	.9140852	.9142383	.9143906	.9148434	.9157300
3 "	.8739367	.8741562	.8743748	.8750243	.8762967
4 "	.8355515	.8358315	.8361102	.8369384	.8385614
5 "	.7988524	.7991870	.7995201	.8005102	.8024511
6 "	.7637652	.7641490	.7645312	.7656676	.7678958
7 "	.7302191	.7306472	.7310735	.7323414	.7348285
8 "	.6981464	.6986142	.6990801	.7004659	.7031852
9 "	.6674824	.6679856	.6684868	.6699777	.6729045
10 "	.6381652	.6386997	.6392323	.6408165	.6439277

5%

To find what sum at 5% per annum, will produce any given amount, interest compounded at tabular intervals.*

See Table No. 22
Page 41

Computed by Formula No. III
Page 123

EXAMPLE

What sum will amount to $2500.00, interest compounded quarterly at 5% per annum for 10 years?

Tabular sum †	=	.6084134	
Amount	=	2500.	
Sum	=	$1521.04	Answer.

* Tabular intervals: Monthly, Bi-monthly, Quarterly, Semi-annually, Annually.

† For tabular sum, see Table No. 22, vertically under quarterly, horizontally from 10 years.

THE PRESENT VALUE OF $1, INTEREST COMPOUNDED AT TABULAR INTERVALS AT 5%

TABLE No. 22 — TABLE No. 22

	Monthly	Bi-monthly	Quarterly	Semi-annually	Annually
1 year.....	.9513284	.9514266	.9515243	.9518144	.9523810
2 years.....	.9050255	.9052125	.9053985	.9059507	.9070295
3 "	.8609764	.8612432	.8615086	.8622969	.8638376
4 "	.8190712	.8194096	.8197464	.8207468	.8227025
5 "	.7792055	.7796080	.7800086	.7811985	.7835262
6 "	.7412802	.7417398	.7421971	.7435559	.7462154
7 "	.7052008	.7057109	.7062186	.7077272	.7106814
8 "	.6708774	.6714321	.6719841	.6736250	.6768394
9 "	.6382247	.6388183	.6394092	.6411659	.6446090
10 "	.6071612	.6077887	.6084134	.6102710	.6139133

$5\frac{1}{2}\%$

To find what sum at $5\frac{1}{2}\%$ per annum, will produce any given amount, interest compounded at tabular intervals.*

See Table No. 23
Page 43

Computed by Formula No. III
Page 123

EXAMPLE

What sum will amount to $9400.00, interest compounded semi-annually at $5\frac{1}{2}\%$ per annum for 8 years?

Tabular sum †	=	.6478743	
Amount	=	9400	
Sum	=	$6090.02	Answer.

* Tabular intervals: Monthly, Bi-monthly, Quarterly, Semi-annually, Annually.

† For tabular sum, see Table No. 23, vertically under semi-annually, horizontally from 8 years.

THE PRESENT VALUE OF $1, INTEREST COMPOUNDED AT TABULAR INTERVALS AT 5½%

TABLE No. 23 — TABLE No. 23

	Monthly	Bi-monthly	Quarterly	Semi-annually	Annually
1 year.....	.9466042	.9467224	.9468399	.9471884	.9478673
2 years.....	.8960594	.8962832	.8965058	.8971658	.8984524
3 "	.8482135	.8485313	.8488474	.8497850	.8516137
4 "	.8029223	.8033236	.8037226	.8049064	.8072168
5 "	.7600496	.7605245	.7609966	.7623980	.7651344
6 "	.7194661	.7200054	.7205419	.7221345	.7252459
7 "	.6810495	.6816452	.6822378	.6839973	.6874369
8 "	.6446843	.6453287	.6459700	.6478743	.6515989
9 "	.6102608	.6109471	.6116301	.6136590	.6176293
10 "	.5776754	.5783973	.5791158	.5812506	.5854306

6%

To find what sum at 6% per annum, will produce any given amount, interest compounded at tabular intervals. *

See Table No. 24
Page 45

Computed by Formula No. III
Page 123

EXAMPLE

What sum will amount to $100,000.00, interest compounded quarterly at 6% per annum for 5 years?

Tabular sum † =	.7424705	
Amount	= 100,000.	
Sum	= $74,247.05	Answer.

* Tabular intervals: Monthly, Bi-monthly, Quarterly, Semi-annually, Annually.

† For tabular sum, see Table No. 24, vertically under quarterly, horizontally from 5 years.

THE PRESENT VALUE OF $1, INTEREST COMPOUNDED AT TABULAR INTERVALS AT 6%

TABLE No. 24 TABLE No. 24

	Monthly	Bi-monthly	Quarterly	Semi-annually	Annually
1 year.....	.9419054	.9420453	.9421843	.9425960	.9433963
2 years.....	.8871858	.8874493	.8877113	.8884871	.8899965
3 "	.8356450	.8360174	.8363875	.8374844	.8396193
4 "	.7870985	.7875662	.7880311	.7894093	.7920937
5 "	.7413723	.7419230	.7424705	.7440940	.7472582
6 "	.6983025	.6989250	.6995440	.7013800	.7049606
7 "	.6577348	.6584190	.6590993	.6611178	.6650572
8 "	.6195239	.6202605	.6209930	.6231670	.6274124
9 "	.5835329	.5843134	.5850898	.5873947	.5918985
10 "	.5496328	.5504496	.5512624	.5536758	.5583948

2%

To find what amount any sum will produce at 2% per annum, successively deposited at tabular intervals, interest compounded at same tabular intervals.*

See Table No. 25
Page 47

Computed by Formula No. IV
Page 125

EXAMPLE

What will $200.00 deposited every month for 5 years amount to at 2% per annum, interest compounded monthly?

Tabular amount †	=	63.152393	
Sum	=	200.	
Amount	=	$12,630.47	Answer.

* Tabular intervals: Monthly, Bi-monthly, Quarterly, Semi-annually, Annually.

† For tabular amount, see Table No. 25, vertically under monthly, horizontally from 5 years.

THE AMOUNT OF $1 AT 2%, SUCCESSIVELY DEPOSITED AT TABULAR INTERVALS, INTEREST COMPOUNDED AT SAME TABULAR INTERVALS

TABLE No. 25 TABLE No. 25

	Monthly	Bi-monthly	Quarterly	Semi-annually	Annually
1 year.....	12.130790	6.070388	4.050250	2.030100	1.020000
2 years.....	24.506432	12.263200	8.182115	4.101005	2.060400
3 "	37.131869	18.580906	12.397240	6.213535	3.121608
4 "	50.012141	25.026023	16.697301	8.368527	4.204040
5 "	63.152393	31.601121	21.084011	10.566834	5.308120
6 "	76.557873	38.308822	25.559115	12.809328	6.434283
7 "	90.233933	45.151799	30.124395	15.096895	7.582969
8 "	104.186036	52.132782	34.781667	17.430443	8.754628
9 "	118.419753	59.254554	39.532786	19.810895	9.949720
10 "	132.940767	66.519952	44.379642	22.239194	11.168715

$2\frac{1}{2}\%$

To find what amount any sum will produce at $2\frac{1}{2}\%$ per annum, successively deposited at tabular intervals, interest compounded at same tabular intervals.*

See Table No. 26
Page 49

Computed by Formula No. IV
Page 125

EXAMPLE

What will $300.00 deposited every two months for 4 years amount to at $2\frac{1}{2}\%$ per annum, interest compounded bi-monthly?

Tabular amount †	=	25.290854
Sum	=	300.
Amount	=	$7587.25 Answer.

* Tabular intervals: Monthly, Bi-monthly, Quarterly, Semi-annually, Annually.

† For tabular amount, see Table No. 26, vertically under bi-monthly, horizontally from 4 years.

THE AMOUNT OF $1 AT 2½%, SUCCESSIVELY DEPOSITED AT TABULAR INTERVALS, INTEREST COMPOUNDED AT SAME TABULAR INTERVALS

TABLE No. 26 TABLE No. 26

	Monthly	Bi-monthly	Quarterly	Semi-annually	Annually
1 year.....	12.163743	6.088108	4.062891	2.037656	1.025000
2 years.....	24.635088	12.330014	8.228310	4.126572	2.075625
3 "	37.421814	18.729601	12.498845	6.268037	3.152515
4 "	50.531898	25.290854	16.877149	8.463374	4.256328
5 "	63.973514	32.017857	21.365941	10.713937	5.387736
6 "	77.755049	38.914796	25.968009	13.021116	6.547430
7 "	91.885096	45.985964	30.686212	15.386334	7.736115
8 "	106.372471	53.235764	35.523480	17.811053	8.954518
9 "	121.226209	60.668707	40.482819	20.296769	10.203381
10 "	136.455575	68.289420	45.567308	22.845015	11.483466

3%

To find what amount any sum will produce at 3% per annum, successively deposited at tabular intervals, interest compounded at same tabular intervals.*

See Table No. 27
Page 51

Computed by Formula No. IV
Page 125

EXAMPLE

What will $425.00 deposited every six months for 7 years amount to at 3% per annum, interest compounded semi-annually?

Tabular amount †	=	15.682137	
Sum	=	425.	
Amount	=	$6664.90	Answer.

* Tabular intervals: Monthly, Bi-monthly, Quarterly, Semi-annually, Annually.

† For tabular amount, see Table No. 27, vertically under semi-annually, horizontally from 7 years.

THE AMOUNT OF $1 AT 3%, SUCCESSIVELY DEPOSITED AT TABULAR INTERVALS, INTEREST COMPOUNDED AT SAME TABULAR INTERVALS

TABLE No. 27 TABLE No. 27

	Monthly	Bi-monthly	Quarterly	Semi-annually	Annually
1 year.....	12.196798	6.105879	4.075564	2.045225	1.030000
2 years.....	24.764575	12.397240	8.274778	4.152266	2.090900
3 "	37.714612	18.879717	12.601393	6.322994	3.183627
4 "	51.058537	25.559115	17.059273	8.559331	4.309135
5 "	64.808330	32.441417	21.652403	10.863262	5.468409
6 "	78.976336	39.532786	26.384884	13.236829	6.662462
7 "	93.575276	46.839573	31.260944	15.682137	7.892336
8 "	108.618256	54.368322	36.284941	18.201355	9.159106
9 "	124.118783	62.125776	41.461361	20.796716	10.463879
10 "	140.090773	70.118882	46.794830	23.470521	11.807795

$3\frac{1}{2}\%$

To find what amount any sum will produce at $3\frac{1}{2}\%$ per annum, successively deposited at tabular intervals, interest compounded at same tabular intervals.*

See Table No. 28
Page No. 53

Computed by Formula No. IV
Page 125

EXAMPLE

What will $15,000.00 deposited annually for 4 years amount to at $3\frac{1}{2}\%$ per annum, interest compounded annually?

Tabular amount †	=	4.362465	
Sum	=	15,000.	
Amount	=	$65,436.97	Answer.

* Tabular intervals: Monthly, Bi-monthly, Quarterly, Semi-annually, Annually.

† For tabular amount, see Table No. 28, vertically under annually, horizontally from 4 years.

THE AMOUNT OF $1 AT 3½%, SUCCESSIVELY DEPOSITED AT TABULAR INTERVALS, INTEREST COMPOUNDED AT SAME TABULAR INTERVALS

TABLE No. 28 TABLE No. 28

	Monthly	Bi-monthly	Quarterly	Semi-annually	Annually
1 year.....	12.229948	6.123696	4.088268	2.052806	1.035000
2 years.....	24.894877	12.464873	8.321515	4.178089	2.106225
3 "	38.010260	19.031252	12.704881	6.378408	3.214942
4 "	51.592116	25.830832	17.243691	8.656412	4.362465
5 "	65.657038	32.871895	21.943456	11.014844	5.550152
6 "	80.222205	40.163017	26.809885	13.456543	6.779407
7 "	95.305412	47.713079	31.848886	15.984449	8.051686
8 "	110.925082	55.531278	37.066582	18.601606	9.368495
9 "	127.100296	63.627134	42.469307	21.311165	10.731393
10 "	143.850813	72.010510	48.063625	24.116389	12.141991

4%

To find what amount any sum will produce at 4% per annum, successively deposited at tabular intervals, interest compounded at same tabular intervals.*

See Table No. 29
Page 55

Computed by Formula No. IV
Page 125

EXAMPLE

What will $940.00 deposited quarterly for 5 years amount to at 4% per annum, interest compounded quarterly?

Tabular amount †	=	22.239194
Sum	=	940.
Amount	=	$20,904.84 Answer.

* Tabular intervals: Monthly, Bi-monthly, Quarterly, Semi-annually, Annually.

† For tabular amount, see Table No. 29, vertically under quarterly, horizontally from 5 years.

THE AMOUNT OF $1 AT 4%, SUCCESSIVELY DEPOSITED AT TABULAR INTERVALS, INTEREST COMPOUNDED AT SAME TABULAR INTERVALS

TABLE No. 29 TABLE No. 29

	Monthly	Bi-monthly	Quarterly	Semi-annually	Annually
1 year.....	12.263200	6.141565	4.101005	2.060400	1.040000
2 years.....	25.026023	12.532923	8.368527	4.204040	2.121600
3 "	38.308822	19.184235	12.809328	6.434283	3.246464
4 "	52.132782	26.106073	17.430443	8.754628	4.416322
5 "	66.519952	33.309440	22.239194	11.168715	5.632975
6 "	81.493277	40.805787	27.243199	13.680331	6.898294
7 "	97.076639	48.607030	32.450387	16.293417	8.214226
8 "	113.294890	56.725570	37.869008	19.012071	9.582795
9 "	130.173898	65.174312	43.507647	21.840558	11.006107
10 "	147.740582	73.966687	49.375237	24.783317	12.486351

4½%

To find what amount any sum will produce at 4½% per annum, successively deposited at tabular intervals, interest compounded at same tabular intervals.*

See Table No. 30
Page 57

Computed by Formula No. IV
Page 125

EXAMPLE

What will $47.50 deposited every two months for 10 years amount to at 4½% per annum, interest compounded bi-monthly?

Tabular amount †	=	75.989817	
Sum	=	47.50	
Amount	=	$3609.51	Answer.

* Tabular intervals: Monthly, Bi-monthly, Quarterly, Semi-annually, Annually.

† For tabular amount, see Table No. 30, vertically under bi-monthly, horizontally from 10 years.

THE AMOUNT OF $1 AT 4½%, SUCCESSIVELY DEPOSITED AT TABULAR INTERVALS, INTEREST COMPOUNDED AT SAME TABULAR INTERVALS

TABLE No. 30

TABLE No. 30

	Monthly	Bi-monthly	Quarterly	Semi-annually	Annually
1 year.....	12.296555	6.159483	4.113772	2.068006	1.045000
2 years.....	25.158011	12.601393	8.415811	4.230119	2.137025
3 "	38.610321	19.338678	12.914734	6.490622	3.278191
4 "	52.680627	26.384884	17.619550	8.853993	4.470709
5 "	67.397321	33.754173	22.539683	11.324911	5.716891
6 "	82.790096	41.461361	27.684985	13.908271	7.019152
7 "	98.890012	49.521941	33.065763	16.609191	8.380013
8 "	115.729556	57.952116	38.692792	19.433019	9.802114
9 "	133.342705	66.768833	44.577343	22.385349	11.288209
10 "	151.764998	75.989817	50.731202	25.472029	12.841178

5%

To find what amount any sum will produce at 5% per annum, successively deposited at tabular intervals, interest compounded at same tabular intervals.*

See Table No. 31
Page 59

Computed by Formula No. IV
Page 125

EXAMPLE

What will $1750.00 deposited every six months for 5 years amount to at 5% per annum, interest compounded semi-annually?

Tabular amount †	=	11.483466
Sum	=	1,750.
Amount	=	$20,096.06 Answer.

* Tabular intervals: Monthly, Bi-monthly, Quarterly, Semi-annually, Annually.

† For tabular amount, see Table No. 31, vertically under semi-annually, horizontally from 5 years.

THE AMOUNT OF $1 AT 5%, SUCCESSIVELY DEPOSITED AT TABULAR INTERVALS, INTEREST COMPOUNDED AT SAME TABULAR INTERVALS

TABLE No. 31

TABLE No. 31

	Monthly	Bi-monthly	Quarterly	Semi-annually	Annually
1 year.....	12.330014	6.177450	4.126572	2.075625	1.050000
2 years.....	25.290854	12.670279	8.463374	4.256328	2.152500
3 "	38.914796	19.494589	13.021116	6.547430	3.310125
4 "	53.235764	26.667302	17.811053	8.954518	4.525631
5 "	68.289420	34.206206	22.845015	11.483466	5.801912
6 "	84.113249	42.129996	28.135435	14.140441	7.142008
7 "	100.746656	50.458322	33.695376	16.931926	8.549108
8 "	118.231058	59.211836	39.538571	19.864730	10.026564
9 "	136.609996	68.412246	45.679449	22.946007	11.577892
10 "	155.929234	78.082368	52.133176	26.183274	13.206787

$5\frac{1}{2}\%$

To find what amount any sum will produce at $5\frac{1}{2}\%$ per annum, successively deposited at tabular intervals, interest compounded at same tabular intervals.*

See Table No. 32
Page 61

Computed by Formula No. IV
Page 125

EXAMPLE

What will $175.00 deposited every month for 8 years amount to at $5\frac{1}{2}\%$ per annum, interest compounded monthly?

Tabular amount †	=	120.801405	
Sum	=	175.	
Amount	=	$21,140.24	Answer.

* Tabular intervals: Monthly, Bi-monthly, Quarterly, Semi-annually, Annually.

† For tabular amount, see Table No. 32, vertically under monthly, horizontally from 8 years.

THE AMOUNT OF $1 AT 5½%, SUCCESSIVELY DEPOSITED AT TABULAR INTERVALS, INTEREST COMPOUNDED AT SAME TABULAR INTERVALS

TABLE No. 32 TABLE No. 32

	Monthly	Bi-monthly	Quarterly	Semi-annually	Annually
1 year.....	12.363575	6.195467	4.139403	2.083256	1.055000
2 years.....	25.424553	12.739590	8.511212	4.282667	2.168025
3 "	39.222273	19.651989	13.128476	6.604708	3.342266
4 "	53.798293	26.953390	18.004975	9.056218	4.581091
5 "	69.196514	34.665684	23.155264	11.644415	5.888051
6 "	85.463316	42.811995	28.594715	14.376921	7.266893
7 "	102.647693	51.416747	34.339563	17.261780	8.721572
8 "	120.801405	60.505739	40.406954	20.307488	10.256258
9 "	139.979128	70.106222	46.814998	23.523014	11.875352
10 "	160.238625	80.246982	53.582820	26.917825	13.583496

6%

To find what amount any sum will produce at 6% per annum, successively deposited at tabular intervals, interest compounded at same tabular intervals.*

See Table No. 33
Page 63

Computed by Formula No. IV
Page 125

EXAMPLE

What will $8250.00 deposited every three months for 2 years amount to at 6% per annum, interest compounded quarterly?

Tabular amount †	=	8.559331
Sum	=	8,250.
Amount	=	$70,614.48 Answer.

* Tabular intervals: Monthly, Bi-monthly, Quarterly, Semi-annually, Annually.

† For tabular amount, see Table No. 33, vertically under quarterly, horizontally from 2 years.

THE AMOUNT OF $1 AT 6%, SUCCESSIVELY DEPOSITED AT TABULAR INTERVALS, INTEREST COMPOUNDED AT SAME TABULAR INTERVALS

TABLE No. 33 TABLE No. 33

	Monthly	Bi-monthly	Quarterly	Semi-annually	Annually
1 year.....	12.397240	6.213535	4.152266	2.090900	1.060000
2 years.....	25.559115	12.809328	8.559331	4.309135	2.183600
3 "	39.532786	19.810895	13.236829	6.662462	3.374616
4 "	54.368322	27.243199	18.201355	9.159106	4.637092
5 "	70.118882	35.132740	23.470521	11.807795	5.975318
6 "	86.840902	43.507647	29.063023	14.617790	7.393837
7 "	104.594299	52.397779	34.998700	17.598913	8.897468
8 "	123.442687	61.834834	41.298611	20.761587	10.491316
9 "	143.453602	71.852457	47.985108	24.116868	12.180794
10 "	164.698747	82.486367	55.081911	27.676485	13.971642

2%

To find what sum at 2% per annum, successively deposited at tabular intervals, will produce any given amount, interest compounded at same tabular intervals.*

See Table No. 34
Page 65

Computed by Formula No. V
Page 128

EXAMPLE

What sum deposited every month for 5 years at 2% per annum will amount to $5000.00, interest compounded monthly?

Tabular sum †	=	.0158347	
Amount	=	5000.	
Sum	=	$ 79.18	Answer.

* Tabular intervals: Monthly, Bi-monthly, Quarterly, Semi-annually, Annually.

† For tabular sum, see Table No. 34, vertically under monthly, horizontally from 5 years.

THE PRESENT VALUE OF $1 AT 2%, SUCCESSIVELY DEPOSITED AT TABULAR INTERVALS, INTEREST COMPOUNDED AT SAME TABULAR INTERVALS

TABLE No. 34 TABLE No. 34

	Monthly	Bi-monthly	Quarterly	Semi-annually	Annually
1 year.....	.0824349	.1647341	.2468984	.4925866	.9803922
2 years.....	.0408056	.0815448	.1222178	.2438427	.4853427
3 "	.0269311	.0538187	.0806631	.1609390	.3203477
4 "	.0199952	.0399584	.0598899	.1194954	.2378665
5 "	.0158347	.0316445	.0474293	.0946358	.1883906
6 "	.0130620	.0261037	.0391250	.0780681	.1554175
7 "	.0110823	.0221475	.0331957	.0662388	.1318745
8 "	.0095982	.0191818	.0287508	.0573709	.1142253
9 "	.0084446	.0168764	.0252955	.0504773	.1005054
10 "	.0075222	.0150331	.0225329	.0449657	.0895358

$2\frac{1}{2}\%$

To find what sum at $2\frac{1}{2}\%$ per annum, successively deposited at tabular intervals, will produce any given amount, interest compounded at same tabular intervals.*

See Table No. 35
Page 67

Computed by Formula No. V
Page 128

EXAMPLE

What sum deposited every two months for 7 years at $2\frac{1}{2}\%$ per annum will amount to $12,500.00, interest compounded bi-monthly?

Tabular sum †	=	.0217458	
Amount	=	12,500.	
Sum	=	$ 271.83	Answer.

* Tabular intervals: Monthly, Bi-monthly, Quarterly, Semi-annually, Annually.

† For tabular sum, see Table No. 35, vertically under bi-monthly, horizontally from 7 years.

THE PRESENT VALUE OF $1 AT 2½%, SUCCESSIVELY DEPOSITED AT TABULAR INTERVALS, INTEREST COMPOUNDED AT SAME TABULAR INTERVALS

TABLE No. 35 TABLE No. 35

	Monthly	Bi-monthly	Quarterly	Semi-annually	Annually
1 year.....	.0822116	.1642547	.2461302	.4907600	.9756098
2 years.....	.0405925	.0811029	.1215317	.2423319	.4817826
3 "	.0267224	.0533915	.0800074	.1595396	.3172071
4 "	.0197895	.0395400	.0592517	.1181562	.2349443
5 "	.0156315	.0312326	.0468035	.0933364	.1856067
6 "	.0128609	.0256972	.0385090	.0767984	.1527317
7 "	.0108832	.0217458	.0325880	.0649928	.1292639
8 "	.0094010	.0187844	.0281504	.0561449	.1116755
9 "	.0082491	.0164830	.0247019	.0492690	.0980068
10 "	.0073284	.0146436	.0219456	.0437733	.0870818

3%

To find what sum at 3% per annum, successively deposited at tabular intervals, will produce any given amount, interest compounded at same tabular intervals.*

See Table No. 36
Page 69

Computed by Formula No. V
Page 128

EXAMPLE

What sum deposited every three months for 9 years at 3% per annum will amount to $54,000.00, interest compounded quarterly?

Tabular sum †	=	.0241189	
Amount	=	54,000.	
Sum	=	$ 1,302.42	Answer.

* Tabular intervals: Monthly, Bi-monthly, Quarterly, Semi-annually, Annually.

† For tabular sum, see Table No. 36, vertically under quarterly, horizontally from 9 years.

THE PRESENT VALUE OF $1 AT 3%, SUCCESSIVELY DEPOSITED AT TABULAR INTERVALS, INTEREST COMPOUNDED AT SAME TABULAR INTERVALS

TABLE No. 36 TABLE No. 36

	Monthly	Bi-monthly	Quarterly	Semi-annually	Annually
1 year.....	.0819888	.1637766	.2453648	.4889438	.9708738
2 years.....	.0403803	.0806631	.1208492	.2408324	.4782630
3 "	.0265150	.0529669	.0793563	.1581530	.3141072
4 "	.0195854	.0391250	.0586192	.1168316	.2320652
5 "	.0154301	.0308248	.0461843	.0920534	.1828686
6 "	.0126621	.0252955	.0379005	.0755468	.1500947
7 "	.0106866	.0213495	.0319888	.0637668	.1267052
8 "	.0092066	.0183931	.0275597	.0549410	.1091810
9 "	.0080568	.0160964	.0241189	.0480845	.0955669
10 "	.0071383	.0142615	.0213699	.0426067	.0846898

$3\frac{1}{2}\%$

To find what sum at $3\frac{1}{2}\%$ per annum, successively deposited at tabular intervals, will produce any given amount, interest compounded at same tabular intervals.*

See Table No. 37
Page 71

Computed by Formula No. V
Page 128

EXAMPLE

What sum deposited every six months for 6 years at $3\frac{1}{2}\%$ per annum will amount to $25,000, interest compounded semi-annually?

Tabular sum †	=	.0743133	
Amount	=	25,000.	
Sum	=	$ 1,857.84	Answer.

* Tabular intervals: Monthly, Bi-monthly, Quarterly, Semi-annually, Annually.

† For tabular sum, see Table No. 37, vertically under semi-annually, horizontally from 6 years.

THE PRESENT VALUE OF $1 AT 3½%, SUCCESSIVELY DEPOSITED AT TABULAR INTERVALS, INTEREST COMPOUNDED AT SAME TABULAR INTERVALS

TABLE No. 37 TABLE No. 37

	Monthly	Bi-monthly	Quarterly	Semi-annually	Annually
1 year.....	.0817665	.1633001	.2446024	.4871381	.9661836
2 years.....	.0401689	.0802255	.1201705	.2393439	.4747831
3 "	.0263087	.0525452	.0787099	.1567790	.3110476
4 "	.0193828	.0387135	.0579923	.1155213	.2292282
5 "	.0152307	.0304212	.0455717	.0907866	.1801753
6 "	.0124654	.0248986	.0372997	.0743133	.1475056
7 "	.0104926	.0209586	.0313983	.0625608	.1241976
8 "	.0090151	.0180079	.0269785	.0537588	.1067408
9 "	.0078678	.0157166	.0235464	.0469238	.0931846
10 "	.0069517	.0138869	.0208058	.0414656	.0823588

4%

To find what sum at 4% per annum, successively deposited at tabular intervals, will produce any given amount, interest compounded at same tabular intervals.*

See Table No. 38
Page 73

Computed by Formula No. V
Page 128

EXAMPLE

What sum deposited every year for 10 years at 4% per annum will amount to $8500.00, interest compounded annually?

Tabular sum †	=	.0800875
Amount	=	8500.
Sum	=	$ 680.75 Answer.

* Tabular intervals: Monthly, Bi-monthly, Quarterly, Semi-annually, Annually.

† For tabular sum, see Table No. 38, vertically under annually, horizontally from 10 years.

THE PRESENT VALUE OF $1 AT 4%, SUCCESSIVELY DEPOSITED AT TABULAR INTERVALS, INTEREST COMPOUNDED AT SAME TABULAR INTERVALS

TABLE No. 38 TABLE No. 38

	Monthly	Bi-monthly	Quarterly	Semi-annually	Annually
1 year.....	.0815448	.1628250	.2438427	.4853427	.9615385
2 years.....	.0399584	.0797899	.1194954	.2378665	.4713424
3 "	.0261037	.0521262	.0780681	.1554175	.3080275
4 "	.0191818	.0383053	.0573709	.1142253	.2264328
5 "	.0150331	.0300215	.0449657	.0895358	.1775261
6 "	.0122710	.0245064	.0367064	.0730977	.1449634
7 "	.0103012	.0205732	.0308163	.0613745	.1217400
8 "	.0088266	.0176288	.0264068	.0525982	.1043537
9 "	.0076821	.0153435	.0229845	.0457864	.0908587
10 "	.0067687	.0135196	.0202531	.0403498	.0800875

$4\frac{1}{2}\%$

To find what sum at $4\frac{1}{2}\%$ per annum, successively deposited at tabular intervals, will produce any given amount, interest compounded at same tabular intervals.*

See Table No. 39
Page 75

Computed by Formula No. V
Page 128

EXAMPLE

What sum deposited every month for 6 years at $4\frac{1}{2}\%$ per annum will amount to $10,000.00, interest compounded monthly?

Tabular sum †	=	.0120788	
Amount	=	10,000.	
Sum	=	$ 120.79	Answer.

* Tabular intervals: Monthly, Bi-monthly, Quarterly, Semi-annually, Annually.

† For tabular sum, see Table No. 39, vertically under monthly, horizontally from 6 years.

THE PRESENT VALUE OF $1 AT 4½%, SUCCESSIVELY DEPOSITED AT TABULAR INTERVALS, INTEREST COMPOUNDED AT SAME TABULAR INTERVALS

TABLE No. 39 TABLE No. 39

	Monthly	Bi-monthly	Quarterly	Semi-annually	Annually
1 year.....	.0813236	.1623513	.2430859	.4835576	.9569378
2 years.....	.0397488	.0793563	.1188240	.2364000	.4679403
3 "	.0258998	.0517099	.0774310	.1540685	.3050463
4 "	.0189823	.0379005	.0567552	.1129434	.2236782
5 "	.0148374	.0296260	.0443662	.0883009	.1749203
6 "	.0120788	.0241189	.0361207	.0718997	.1424674
7 "	.0101123	.0201931	.0302428	.0602077	.1193316
8 "	.0086409	.0172557	.0258446	.0514588	.1020188
9 "	.0074995	.0149771	.0224330	.0446721	.0885880
10 "	.0065892	.0131597	.0197118	.0392588	.0778745

5%

To find what sum at 5% per annum, successively deposited at tabular intervals, will produce any given amount, interest compounded at same tabular intervals.*

See Table No. 40 Page 77

Computed by Formula No. V Page 128

EXAMPLE

What sum deposited every two months for 8 years at 5% per annum will amount to $1750.00, interest compounded bi-monthly?

Tabular sum †	=	.0168885	
Amount	=	1750.	
Sum	=	$ 29.56	Answer.

* Tabular intervals: Monthly, Bi-monthly, Quarterly, Semi-annually, Annually.

† For tabular sum, see Table No. 40, vertically under bi-monthly, horizontally from 8 years.

THE PRESENT VALUE OF $1 AT 5%, SUCCESSIVELY DEPOSITED AT TABULAR INTERVALS, INTEREST COMPOUNDED AT SAME TABULAR INTERVALS

TABLE No. 40 TABLE No. 40

	Monthly	Bi-monthly	Quarterly	Semi-annually	Annually
1 year.....	.0811029	.1618791	.2423319	.4817826	.9523810
2 years.....	.0395400	.0789249	.1181562	.2349443	.4645761
3 "	.0256972	.0512963	.0767984	.1527317	.3021034
4 "	.0187844	.0374991	.0561449	.1116755	.2209637
5 "	.0146436	.0292345	.0437733	.0870818	.1723570
6 "	.0118888	.0237361	.0355424	.0707192	.1400167
7 "	.0099259	.0198184	.0296777	.0590601	.1169713
8 "	.0084580	.0168885	.0252918	.0503405	.0997351
9 "	.0073201	.0146173	.0218917	.0435806	.0863715
10 "	.0064132	.0128070	.0191817	.0381924	.0757187

$5\frac{1}{2}\%$

To find what sum at $5\frac{1}{2}\%$ per annum, successively deposited at tabular intervals, will produce any given amount, interest compounded at same tabular intervals.*

See Table No. 41
Page 79

Computed by Formula No. V
Page 128

EXAMPLE

What sum deposited every three months for 4 years at $5\frac{1}{2}\%$ per annum will amount to $18,000.00, interest compounded quarterly?

Tabular sum †	=	.0555402	
Amount	=	18,000.	
Sum	=	$ 999.73	Answer.

* Tabular intervals: Monthly, Bi-monthly, Quarterly, Semi-annually, Annually.

† For tabular sum, see Table No. 41, vertically under quarterly, horizontally from 4 years.

THE PRESENT VALUE OF $1 AT 5½%, SUCCESSIVELY DEPOSITED AT TABULAR INTERVALS, INTEREST COMPOUNDED AT SAME TABULAR INTERVALS

TABLE No. 41 TABLE No. 41

	Monthly	Bi-monthly	Quarterly	Semi-annually	Annually
1 year.....	.0808828	.1614084	.2415808	.4800178	.9478673
2 years.....	.0393321	.0784955	.1174921	.2334994	.4612493
3 "	.0254957	.0508855	.0761703	.1514072	.2991982
4 "	.0185880	.0371011	.0555402	.1104214	.2182886
5 "	.0144516	.0288470	.0431868	.0858781	.1698355
6 "	.0117010	.0233580	.0349715	.0695560	.1376104
7 "	.0097421	.0194489	.0291210	.0579315	.1146583
8 "	.0082781	.0165274	.0247482	.0492429	.0975015
9 "	.0071440	.0142641	.0213607	.0425116	.0842081
10 "	.0062407	.0124616	.0186627	.0371501	.0736188

6%

To find what sum at 6% per annum, successively deposited at tabular intervals, will produce any given amount, interest compounded at same tabular intervals.*

See Table No. 42
Page 81

Computed by Formula No. V
Page 128

EXAMPLE

What sum deposited every six months for 10 years at 6% per annum will amount to $37,500.00, interest compounded semi-annually?

Tabular sum †	=	.0361318	
Amount	=	37,500.	
Sum	=	$ 1,354.95	Answer.

* Tabular intervals: Monthly, Bi-monthly, Quarterly, Semi-annually, Annually.

† For tabular sum, see Table No. 42, vertically under semi-annually, horizontally from 10 years.

THE PRESENT VALUE OF $1 AT 6%, SUCCESSIVELY DEPOSITED AT TABULAR INTERVALS, INTEREST COMPOUNDED AT SAME TABULAR INTERVALS

TABLE No. 42 TABLE No. 42

	Monthly	Bi-monthly	Quarterly	Semi-annually	Annually
1 year.....	.0806631	.1609390	.2408324	.4782630	.9433963
2 years.....	.0391250	.0780681	.1168316	.2320652	.4579594
3 "	.0252955	.0504773	.0755468	.1500947	.2963300
4 "	.0183931	.0367064	.0549410	.1091810	.2156524
5 "	.0142615	.0284635	.0426067	.0846898	.1673551
6 "	.0115153	.0229845	.0344080	.0684098	.1352478
7 "	.0095608	.0190848	.0285725	.0568217	.1123916
8 "	.0081010	.0161721	.0242139	.0481659	.0953170
9 "	.0069709	.0139174	.0208398	.0414648	.0820965
10 "	.0060717	.0121232	.0181548	.0361318	.0715736

TIME AND RATE REQUIRED FOR THE AMOUNT TO EQUAL THE TABULATED INCREASE OF ANY SUM, INTEREST COMPOUNDED ANNUALLY

COMPUTED BY FORMULA No. VI, PAGE 130

TABLE No. 43 — TABLE No. 43

Rate	Increase 10% ($1\frac{1}{10}$)			Increase 25% ($1\frac{1}{4}$)			Increase 50% ($1\frac{1}{2}$)			Increase 100% (Double)		
Per cent.	yrs.	mos.	days	yrs.	mos.	days	yrs.	mos.	days	yrs.	mos.	days
1	9	6	29	22	5	4	40	9	0	69	7	28
$1\frac{1}{2}$	6	4	25	14	11	26	27	2	24	46	6	20
2	4	9	23	11	3	7	20	5	21	35	0	1
$2\frac{1}{2}$	3	10	9	9	0	14	16	5	2	28	0	26
3	3	2	21	7	6	18	13	8	18	23	5	12
$3\frac{1}{2}$	2	9	8	6	5	25	11	9	13	20	1	24
4	2	5	5	5	8	9	10	4	2	17	8	3
$4\frac{1}{2}$	2	2	0	5	0	25	9	2	16	15	8	29
5	1	11	14	4	6	27	8	3	22	14	2	15
$5\frac{1}{2}$	1	9	11	4	2	1	7	6	27	12	11	11
6	1	7	19	3	9	29	6	11	15	11	10	23
$6\frac{1}{2}$	1	6	5	3	6	16	6	5	8	11	0	3
7	1	4	27	3	3	18	5	11	28	10	2	28
$7\frac{1}{2}$	1	3	25	3	1	1	5	7	9	9	7	1
8	1	2	26	2	10	24	5	3	7	9	0	3

AREA

Units of Surface

Square inch (as shown by graph)

Square foot = 144 sq. ins.
Acre = 43,560 sq. ft.
Square mile = 640 acres.
Square mile = 27,878,400 sq. ft.

Formula No. VIII, *a*, *b*, *c*, *d*, *e*, *f*

(*a*) Acres × 43,560 = No. of sq. feet.

(*b*) Square miles × 27,878,400 = No. of sq. feet.

(*c*) Square feet ÷ 43,560 = No. of acres.

(*d*) Square miles × 640 = No. of acres.

(*e*) Square feet ÷ 27,878,400 = No. of sq. miles.

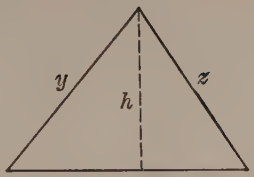
(*f*) Acres ÷ 640 = No. of sq. miles.

The authorized standard of measurement in the United States is the yard, from which the inch is obtained by taking 1 inch = $\frac{1}{36}$ yard.

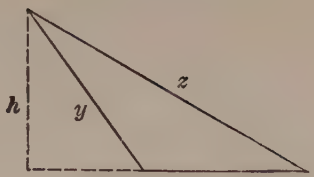
SYMBOLS

		Example.
$\therefore$	therefore.	
$+$	plus (addition)	$(a+b)$.
$-$	minus (subtraction)	$(a-b)$.
$\times$	times (multiplication)	$a \times b$, or ab.
$\div$	divided by (division)	$a \div b$ or $\frac{a}{b}$.
$=$	is equal to	$2a=b$.

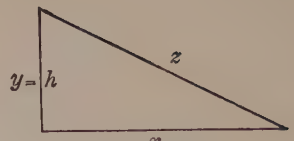
		Example.
$\sqrt{\ }$	square root	$\sqrt{25}=5$.
. . .	and so on	1, 2, 3, 4 . . .
$(\ )$	three signs of aggregation	$(a+b)c=ac+bc$.
$[\ ]$		$a-[b-c]=a-b+c$.
$\{ \ \}$		$\{a+b\} \div c=\frac{a}{c}+\frac{b}{c}$.



Acute (1)



Obtuse (2)



Right (3)

Area $= \frac{1}{2}xh$ (one-half the product of the base and altitude), No. IX.

Where h is not known and the three sides are given.

Area $= \sqrt{s(s-x)(s-y)(s-z)}$ [s being $\frac{1}{2}(x+y+z) = \frac{1}{2}$ the perimeter], No. X, and is read, Area is equal to the square root of the product of s, $s-x$, $s-y$, $s-z$.

EXAMPLE

In triangle acute (1) if $x=13$, $y=14$, $z=15$, then $s=\frac{1}{2}(13+14+15)=21$, $s-x=8$, $s-y=7$, $s-z=6$.

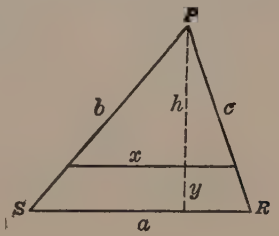
$$\therefore \quad \text{Area} = \sqrt{21\times8\times7\times6} = \sqrt{7056} = 84.$$

In triangle obtuse (2) if $x=10$, $h=12$, area $=\frac{1}{2}(10\times12)=60$.

In triangle right (3) $x^2+y^2=z^2$ or $z=\sqrt{x^2+y^2}$ if $x=4$, $y=3$, $z=\sqrt{16+9}=5$.

To divide a triangle into two equivalent areas, the divisional line being parallel to any side.

One area will be a trapezoid, the other a triangle.



In the triangle PSR, let a be the base, h the altitude and x the divisional line parallel to base.

To find y, the perpendicular distance of x from a, in terms of h.

$\frac{1}{2}x(h-y)=\frac{1}{2}(x+a)y$,

$hx-xy=xy+ay$,

$hx-2xy=ay$,

$x(h-2y)=ay$,

$$x=\frac{ay}{h-2y}, \quad (1)$$

$\frac{1}{2}(x+a)y+\frac{1}{2}x(h-y)=\frac{1}{2}ah$,

$xy+ay+hx-xy=ah$,

$hx=ah-ay$,

$$x=\frac{a(h-y)}{h}, \quad (2)$$

(1) = (2) $\frac{a(h-y)}{h}=\frac{ay}{h-2y}$,

$h^2-3hy+2y^2=hy$,

$h^2-4hy=-2y^2$,

$h^2-4hy+4y^2=2y^2$,

$h-2y=y\sqrt{2}$,

$h=y(2+\sqrt{2})$,

$$y=\frac{h}{2+\sqrt{2}}$$

$y=\frac{h}{2}(2-\sqrt{2})$,

$\therefore$ **y = .29289325h.** (No. XI)

NOTE. $\sqrt{2}=1.4142135$

Therefore, multiply the given altitude by the decimal .29289325 to find the distance of the divisional line from the base, so that the triangle may be divided into two equivalent areas.

EXAMPLE

Divide the triangle whose altitude is 6 into two equivalent areas by divisional line parallel to base.

$y=6\times.29289325=1.7573595$. Answer.

NOTE.—The altitude of the small triangle in this case is $6-1.7573595=4.2426405$.

The length of divisional line, or base of small triangle, is $\frac{a}{\sqrt{2}}$, which equals the base of large triangle divided by 1.4142135. Proof: $2xy+ay=hx$.

$$y=\frac{hx}{2x+a}=\frac{h}{2+\sqrt{2}} \quad \text{or} \quad \frac{x}{2x+a}=\frac{1}{2+\sqrt{2}},$$

$$2x+x\sqrt{2}=2x+a,$$

$$x\sqrt{2}=a,$$

$$\therefore \quad x=\frac{a}{\sqrt{2}}.$$

The altitude of the small triangle is $\frac{h}{\sqrt{2}}$, which equals the altitude of the large triangle divided by 1.4142135.

Proof: $h-y=h-\frac{h(2-\sqrt{2})}{2}=\frac{2h-2h+h\sqrt{2}}{2}=\frac{h\sqrt{2}}{2}=\frac{h}{\sqrt{2}}$.

To find the length of the divisional line the length of the base line of the large triangle must be known.

Any side of the large triangle may be regarded as the base line, but the altitude in each instance is the perpendicular distance from the base to the vertex as shown in figures *b* and *c*.

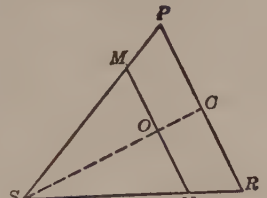


Figure *b*

PR=base:
SC=altitude;
MN=divisional line;
OC=required distance of *MN* from *PR*;
S is the vertex.

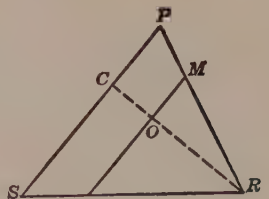
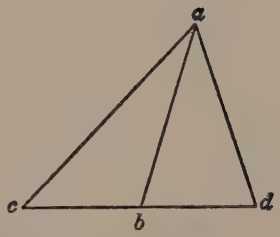
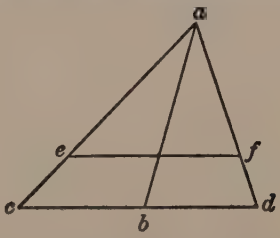


Figure *c*

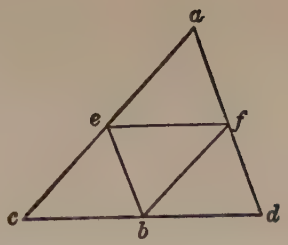
PS=base;
RC=altitude;
MN=divisional line;
OC=required distance of *MN* from *PS*;
R is the vertex.



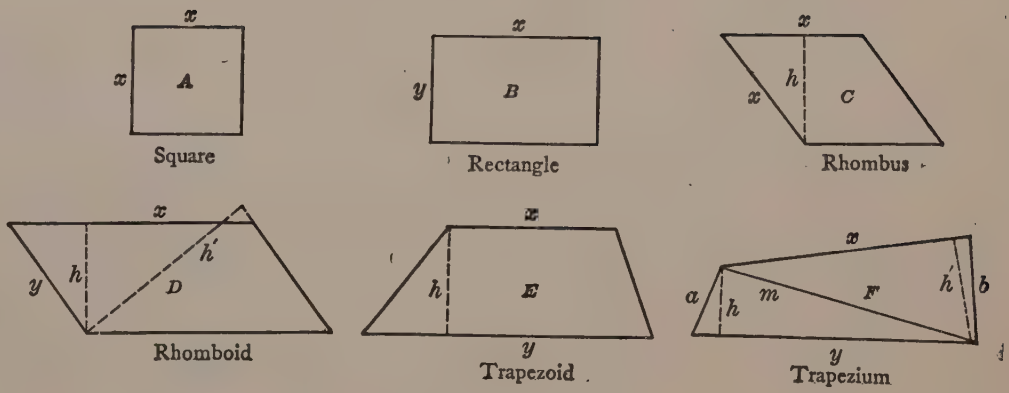
The medial line (*ab*) drawn from the vertex *a*, to *b*, the middle point of the base (*cd*), divides the triangle into two equivalent areas.



The line (*ef*), drawn parallel to base (*cd*), dividing the triangle into two equivalent areas and the medial line (*ab*), divides the triangle into four equivalent areas.



The lines joining the middle points of a triangle as (*ef*), (*eb*) and (*bf*), forming the triangle (*bef*), divides the triangle (*acd*) into four equal areas.



THE SQUARE. (*A*)

Area $= x^2$ (the product of any two sides). No. XII.

To divide a square into two equal areas—Draw either one diameter or one diagonal.

To divide a square into four equal areas—Draw either two diameters or two diagonals.

To divide a square into eight equal areas—Draw two diameters and two diagonals.

The Rectangle. (*B*)

Area = **xy** (the product of any two adjacent sides). No. XIII.

To divide a rectangle into two, four or eight equal areas. Same process as in the square.

The Rhombus. (*C*)

Area = **xh** (the product of any side, and the altitude). No. XIV.

To divide a rhombus into two, four or eight equivalent areas. Same process as in the square.

The Rhomboid. (*D*)

Area = **xh** = **yh′** (the product of any side and corresponding altitude). No. XV.

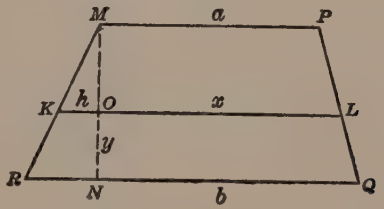
To divide a rhomboid into two, four or eight equivalent areas. Same process as in the square.

The Trapezoid. (*E*)

Area $=\frac{1}{2}(x+y)h$ (the product of one-half the sum of the parallel sides and the altitude). No. XVI.

To divide a trapezoid into two equivalent areas, the divisional line being parallel to the parallel sides.

Both areas will be trapezoids.



In the trapezoid $MPQR$, let $MP=a$ and $RQ=b$ be its parallel sides, $MN=h$ the altitude and $KL=x$ the divisional line parallel to a and b.

To find $ON=y$, the perpendicular distance of x from b, in terms of a, b and h.

$$\text{Area } MPQR=\tfrac{1}{2}(a+b)h,$$

Area $KLQR = \frac{1}{2}(x+b)y$,

Area $MPLK = \frac{1}{2}(x+a)(h-y)$.

$\frac{1}{2}(x+b)y = \frac{1}{2}(x+a)(h-y)$, . (1)

$xy+by = xh-xy+ah-ay$,

$2xy-xh = ah-ay-by$,

$x(2y-h) = ah-ay-by$,

$$x = \frac{ah-ay-by}{2y-h}. \quad . \quad . \quad (2)$$

$\frac{1}{2}(x+b)y+\frac{1}{2}(x+a)(h-y) = \frac{1}{2}(a+b)h$, (3)

$xy+by+xh-xy+ah-ay = ah+bh$,

$xh = bh+ay-by$

$$x = \frac{bh+ay-by}{h}, \quad . \quad . \quad . \quad (4)$$

$(2) = (4) \quad \therefore \quad \frac{bh+ay-by}{h} = \frac{ah-ay-by}{2y-h}$,

$2bhy+2ay^2-2by^2-bh^2-ahy+bhy = ah^2-ahy-bhy$,

$2ay^2-2by^2+4bhy = ah^2+bh^2$,

$(2a-2b)y^2+4bhy = h^2(a+b)$,

$$y^2+\frac{2bh}{(a-b)}y = \frac{h^2(a+b)}{2(a-b)},$$

$$y^2+\frac{2bh}{(a-b)}y+\frac{b^2h^2}{(a-b)^2} = \frac{h^2(a+b)}{2(a-b)}+\frac{b^2h^2}{(a-b)^2},$$

$$\left(y+\frac{bh}{a-b}\right)^2 = \frac{h^2(a^2+b^2)}{2(a-b)^2},$$

$$y+\frac{bh}{a-b} = \frac{h\sqrt{a^2+b^2}}{\sqrt{2}(a-b)},$$

$$y=\frac{h}{\sqrt{2}}\,\frac{\sqrt{a^2+b^2}}{(a-b)}-\frac{bh}{a-b},$$

$$\mathbf{y=\frac{h}{a-b}\left(\sqrt{\frac{a^2+b^2}{2}}-b\right)}$$

No. XVII (*a*)

If b be greater than a

$$\mathbf{y=\frac{h}{b-a}\left(b-\sqrt{\frac{a^2+b^2}{2}}\right)}$$

No. XVII (*b*)

y, the distance of the divisional line x from the lower base (b), is equal to the quotient of the altitude and the difference of the bases, multiplied by the difference of the lower base and the square root of one-half of the sum of the squares of the bases.

EXAMPLE

Divide the trapezoid whose upper base is 6, lower base 8, and altitude 5, into two equivalent areas, by divisional line parallel to bases. Here $a=6$, $b=8$ and $h=5$. Let x be the divisional line.

To find y, the distance of x from b.

Apply Formula No. XVII (*b*):

$$y=\frac{h}{b-a}\left(b-\sqrt{\frac{a^2+b^2}{2}}\right),$$

$$y=\frac{5}{2}\left(8-\sqrt{\frac{36+64}{2}}\right)=\frac{5}{2}\left(8-\frac{10}{\sqrt{2}}\right)=20-\frac{25}{\sqrt{2}},$$

$$y=20-\frac{25}{\sqrt{2}}=20-17.67766875.$$

$\therefore$ $y=2.32233125$ (the altitude of lower trapezoid). **Answer.**

$h-y=5-2.32233125=2.67766875$ (the altitude of upper trapezoid).

$\therefore$ To divide the trapezoid into two equivalent areas, draw the divisional line parallel to a and b, at a distance 2.32233125 from b and 2.67766875 from a.

To find the length of the divisional line x, in terms of the bases (a and b). Eliminate y in the equations,

$$2xy-xh=ah-ay-by. \quad . \quad . \quad (1)$$

$$xh=ay+bh-by. \quad . \quad . \quad (2)$$

(1)

$$2xy+ay+by=ah+xh,$$

$$y=\frac{(a+x)h}{2x+a+b}. \quad . \quad (3)$$

(2)

$$ay-by=xh-bh$$

$$y=\frac{(x-b)h}{a-b}. \quad . \quad . \quad (4)$$

(4) = (3) $\quad \dfrac{(x-b)h}{a-b}=\dfrac{(a+x)h}{2x+a+b}$,

divide by h,

$$\frac{x-b}{a-b}=\frac{a+x}{2x+a+b},$$

$$2x^2-2bx+ax-ab+bx-b^2 = a^2+ax-ab-bx,$$

$$2x^2=a^2+b^2,$$

$$x^2=\frac{a^2+b^2}{2},$$

$$\mathbf{x=\sqrt{\frac{a^2+b^2}{2}}}. \quad . \quad . \quad (5)$$

Therefore the length of the divisional line x equals the square root of one-half of the sum of the squares of the bases.

If $a=3$ and $b=4$, then from (5),

$$x=\sqrt{\frac{9+16}{2}}=\frac{5}{\sqrt{2}}=\frac{5}{2}\sqrt{2}.$$

five halves of the square root of 2.

If $h=2$ then from (4),

$$y=\left(\frac{\frac{5}{2}\sqrt{2}-4}{-1}\right)2=2\left(\frac{4-\frac{5}{2}\sqrt{2}}{1}\right).$$

$$\therefore \quad y=8-5\sqrt{2}$$

and $h-y=2-(8-5\sqrt{2})=5\sqrt{2}-6$.

Scholium or Notation

$y<(h-y)$, y is less than $(h-y)$.

$$\therefore \quad 8-5\sqrt{2}<5\sqrt{2}-6,$$

$$14<10\sqrt{2}.$$

Squaring $196<200$ (proof).

Therefore the altitude of the lower trapezoid is less than the altitude of the upper trapezoid, that is, x is nearer to b than it is to a.

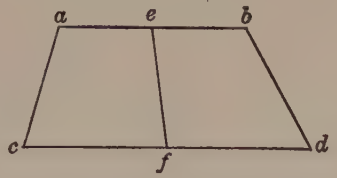
$x > \frac{1}{2}(a+b)$, x is greater than $\frac{1}{2}(a+b)$. Squaring $50 > 49$ (proof).

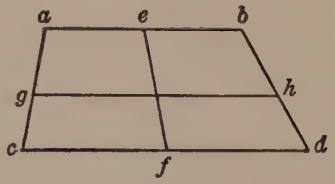
$\therefore \quad \frac{5}{2}\sqrt{2} > \frac{1}{2}(3+4)$.

$5\sqrt{2} > 7$.

Therefore the divisional line is greater than one-half the sum of the bases.

The line (*ef*) joining the middle points of the parallel sides of a trapezoid divides the trapezoid into two equivalent areas.





The line (*gh*) drawn parallel to base (*cd*) dividing the trapezoid into two equivalent areas and the line (*ef*) joining the middle points of the parallel sides divides the trapezoid (*abcd*) into four equivalent areas.

The Trapezium. (*F.*)

Area = triangle amy + triangle xmb.

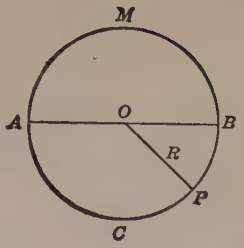
Area $= \frac{1}{2}yh + \frac{1}{2}xh'$ (the sum of the areas of the two triangles of the trapezium) No. XVIII.

If h and h' are not known, a, y, m, x and b must be given to effect a solution.

Area $= \sqrt{s(s-a)(s-m)(s-y)} + \sqrt{s'(s'-x)(s'-m)(s'-b)}$ (the sum of the areas of the two triangles of the trapezium), where $s = \frac{1}{2}(a+m+y)$ and $s' = \frac{1}{2}(x+m+b)$.

Note.—See solution of triangles, page 85.

$AB = D$ (a diameter).
$OP = R$ (a radius).
$ACBM = C$ (the circumference).
O represents the center.



$D = 2R$.

$\frac{C}{D} = \pi(\text{pi}) = 3.1416$.

Area $= \pi R^2$ (the product of 3.1416 and the square of the radius). No. XIX.

EXAMPLE

What is the area of the circle whose radius is 5?

$$\text{Area} = \pi R^2 = 3.1416 \times 25 = 78.54. \quad \text{Answer.}$$

NOTE.—The area of any sector is to the area of the circle as its arc is to the circumference.

EXAMPLE

If arc $BP = \frac{1}{8}$ of C, area of sector $BOP = \frac{78.54}{8} = 9.8175$.

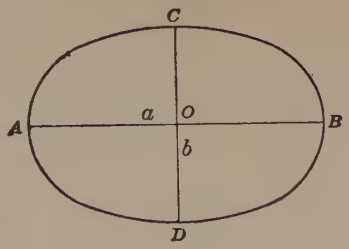
To divide a circle into two equal areas.—Draw a diameter.

To divide a circle into three equal areas.—Apply the radius six times to the circumference and draw radii to every other point of division.

To divide a circle into four equal areas.—Draw two perpendicular diameters.

To divide a circle into six equal areas. —Apply the radius six times to the circumference and draw radii to every point of division.

AB = the major axis = $2a$.
CD = the minor axis = $2b$.
O represents the center.
$AO = a$, $DO = b$.
AB is perpendicular to CD.



Area = πab (the product of 3., 1416 one-half the major axis and one-half the minor axis.) No. XX.

EXAMPLE

What is the area of the ellipse whose major axis is 10 and minor axis 6?

Area = $\pi ab = 3.1416 \times 5 \times 3 = 47.124$. Answer.

To divide an ellipse in two equal areas.—Draw a straight line through the intersection of the axes terminating in the curve.

To divide an ellipse into four equal areas.—Draw the major and minor axes.

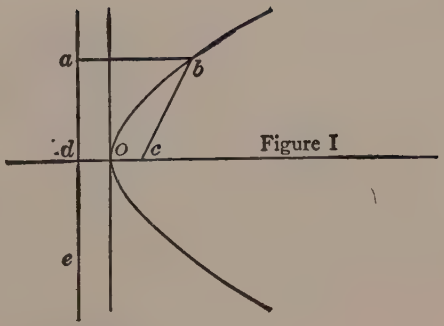


Figure I

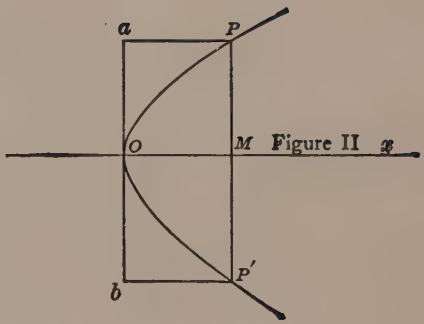


Figure II

$ab = bc.$

$do = oc.$

An open curve generated by a point moving in the same plane so as to remain always equidistant from a fixed point and a fixed line, i.e., any point on the curve as b, is the same distance from point c as from the line ae. (Figure I.)

The area of segment POP' cut off by the chord PP' perpendicular to the axis ox, is $\frac{2}{3}$ **of the rectangle PabP′.** No. XXI.

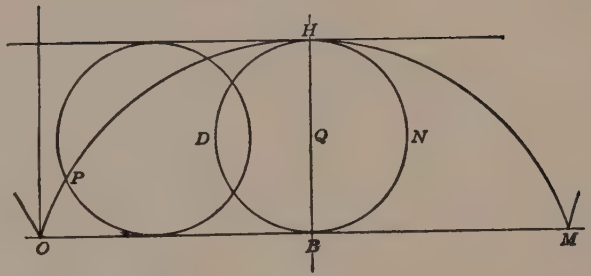
EXAMPLE

In Fig. II what is the area of the segment POP' if $ab = PP' = 5$ and $aP = bP' = 2$?

$$\text{Area} = \frac{2}{3}(5 \times 2) = \frac{20}{3} = 6\tfrac{2}{3}. \quad \text{Answer.}$$

NOTE.—OM divides the segment POP' into the two equal areas POM and OMP'.

THE CYCLOID



A curve generated by a point on the circumference of a circle as it rolls along a straight line. (In the figure *OHM*.)

Area *OPHDBO* = Area *HMBNH* = Area of circle *HNBDH* = πR^2.

$\pi R^2 = 3.1416 \times \overline{HQ}^2$ (the product of 3.1416 and the square of the radius of the circle *HNBDH*).

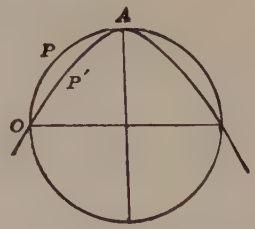
$\therefore$ Area of Cycloid **OHMO** = $3\pi R^2$ (three times the product of 3.1416 and the square of the radius of the generating circle.) No. XXII

EXAMPLE

What is the area of the cycloid whose generating circle has a radius of 4?

$$\text{Area} = 3\pi R^2 = 3 \times 3.1416 \times 16 = 150.7968. \quad \text{Answer.}$$

The area between the parabola and the circle where the axis of the parabola coincides with a diameter of the circle which is perpendicular to the diameter which joins the intersections of the two curves.



$$\text{Area} \quad OPAP' = \frac{\pi R^2}{4} - \frac{2}{3}R^2,$$

one-fourth the product of 3.1416 and the square of the radius, diminished by two-thirds of the square of the radius. No. XXIII.

EXAMPLE

Find the area between the parabola and circle as shown in the figure, the radius of the circle being 3.

$$\text{Area } OPAP' = \frac{3.1416 \times 9}{4} - \frac{2}{3} \times 9 = 7.0686 - 6.$$

$$\therefore \text{ Area } OPAP' = 1.0686. \quad \text{Answer.}$$

To find the proportional holdings where one or more members sell to the remaining members, or where one or more additional members purchase interests in the syndicate.

Suppose a syndicate owning x shares is composed of A, B, C, D and E, owning a, b, c, d and e shares respectively.

CASE I (a). Where one member, say E, sells to the remaining members, and the total number of shares is reduced from x to $x-e$, then the four remaining members will own the same number of shares as originally.

EXAMPLE

A, B, C, D, and E compose a syndicate owning property in given proportions. A owns 6 shares, B owns 4 shares, C owns 15 shares, D owns 10 shares, E owns 1 share. E's interest is absorbed by A, B, C and D in proportion to their holdings. Find A, B, C and D's holdings after E's interest has been acquired.

Here $a=6$, $b=4$, $c=15$, $d=10$, $e=1$, $x=36$, $x-e=35$.

$\therefore$ A owns 6 shares.

B owns 4 shares,

C owns 15 shares,

D owns 10 shares,

Total = 35 shares.

CASE I (*b*). Where E's shares are not cancelled, but proportionally distributed among the remaining members, then

A owns $\frac{ax}{x-e}$ shares, B owns $\frac{bx}{x-e}$ shares, C owns $\frac{cx}{x-e}$ shares, D owns $\frac{dx}{x-e}$ shares,

No. XXIV

then

$$\text{A owns } \frac{6\times 36}{35} = 6\tfrac{6}{35} \text{ shares,}$$

$$\text{B owns } \frac{4\times 36}{35} = 4\tfrac{4}{35} \text{ shares,}$$

$$\text{C owns } \frac{15\times36}{35}=15\tfrac{15}{35} \text{ shares,}$$

$$\text{D owns } \frac{10\times36}{35}=10\tfrac{10}{35} \text{ shares,}$$

$$\text{Total}=36 \text{ shares.}$$

Case II (a). Where more than one member, say D and E, sell to the remaining members: If D's and E's shares are cancelled and the number of shares is reduced from x to $(x-d-e)$, the three remaining members will own the same number of shares as originally.

Here $a=6$, $b=4$, $c=15$, $x-d-e=25$.

$\therefore$ A owns 6 shares,

B owns 4 shares,

C owns 15 shares,

Total = 25 shares.

Case II (*b*). If D's and E's shares are not cancelled, but proportionally distributed among the remaining members, then

$$\text{A owns } \frac{ax}{x-d-e} \text{ shares, B owns } \frac{bx}{x-d-e} \text{ shares, C owns } \frac{cx}{x-d-e} \text{ shares.}$$

No. XXV.

$$\therefore \quad \text{A owns } \frac{6\times 36}{25} = 8\tfrac{16}{25} \text{ shares,}$$

$$\text{B owns } \frac{4\times 36}{25} = 5\tfrac{19}{25} \text{ shares,}$$

$$\text{C owns } \frac{15\times 36}{25} = 21\tfrac{15}{25} \text{ shares.}$$

$$\text{Total} = 36 \text{ shares.}$$

Case III (*a*). Where one additional member, say F, is added to the syndicate and the total number of shares is increased by f shares, then $x+f=$ the total number

of shares and the five members, A, B, C, D and E, will own the same number of shares as originally.

EXAMPLE

Suppose F is alloted a $\frac{1}{10}$ interest, then $\frac{f}{x+f}=\frac{1}{10}$ or $10f=x+f$, $9f=x=36$.

$\therefore$ $f=4$.

Here $a=6, b=4, c=15, d=10, e=1, f=4, x+f=40$.

$\therefore$ A owns 6 shares,
B owns 4 shares,
C owns 15 shares,
D owns 10 shares,
E owns 1 share,
F owns 4 shares.

Total=40 shares.

Case III (*b*). Where an additional member, say F, is added to the syndicate and the total number of shares remains the same, then

A owns $\frac{ax}{x+f}$ shares, B owns $\frac{bx}{x+f}$ shares, C owns $\frac{cx}{x+f}$ shares, D owns $\frac{dx}{x+f}$ shares, E owns $\frac{ex}{x+f}$ shares, F owns $\frac{fx}{x+f}$ shares. No. XXVI.

$$\therefore \quad \text{A owns } \frac{6\times36}{36+4}=\frac{216}{40}= 5\tfrac{16}{40} \text{ shares,}$$

$$\text{B owns } \frac{4\times36}{36+4}=\frac{144}{40}= 3\tfrac{24}{40} \text{ shares,}$$

$$\text{C owns } \frac{15\times36}{36+4}=\frac{540}{40}=13\tfrac{20}{40} \text{ shares,}$$

$$\text{D owns } \frac{10\times36}{36+4}=\frac{360}{40}= 9 \text{ shares,}$$

$$\text{E owns } \frac{1\times36}{36+4}=\frac{36}{40}=\frac{36}{40} \text{ share,}$$

F owns $\frac{4\times 36}{36+4}=\frac{144}{40}=3\frac{24}{40}$ shares.

Total = 36 shares.

CASE IV (*a*). Where more than one member, say F and G, are added to the syndicate and the total number of shares is increased by $(f+g)$ shares, then $(x+f+g)$ = the total number of shares and the five members (A, B, C, D and E) will own the same number of shares as originally.

EXAMPLE

Suppose F purchases a $\frac{3}{41}$ interest, and G purchases a $\frac{2}{41}$ interest then

$$\frac{f}{x+f+g}=\frac{3}{41} \text{ or } \frac{f}{36+f+g}=\frac{3}{41} \text{ and } \frac{g}{x+f+g}=\frac{2}{41} \text{ or } \frac{g}{36+f+g}=\frac{2}{41}.$$

$$41f=108+3f+3g, \qquad 41g=72+2f+2g,$$
$$38f=108+3g, \qquad 39g=72+2f \qquad (n)$$
$$3g=38f-108,$$
$$39g=494f-1404 \quad (m).$$

$(m)=(n)$ $\therefore$ $494f-1404=72+2f,$

$492f=1476,$

$f=3.$

Substitute in (n) the value of f, then

$39g=72+6=78.$ $\therefore$ $g=2.$

Here $a=6,\ b=4,\ c=15,\ d=10,\ e=1,\ f=3,\ g=2,\ x+f+g=41.$

$\therefore$ A owns 6 shares,
B owns 4 shares,
C owns 15 shares,
D owns 10 shares,
E owns 1 share,
F owns 3 shares,
G owns 2 shares,

Total = 41 shares.

CASE IV (*b*). Where more than one member, say F and G, are added to the syndicate and the total number of shares remains the same, then

A owns $\frac{ax}{x+f+g}$ shares, B owns $\frac{bx}{x+f+g}$ shares, C owns $\frac{cx}{x+f+g}$ shares, D owns $\frac{dx}{x+f+g}$ shares, E owns $\frac{ex}{x+f+g}$ shares, F owns $\frac{fx}{x+f+g}$ shares. G owns $\frac{gx}{x+f+g}$ shares.

No. XXVII.

$\therefore$ A owns $\frac{6\times36}{41}=\frac{216}{41}=5\frac{11}{41}$ shares,

B owns $\frac{4\times36}{41}=\frac{144}{41}=3\frac{21}{41}$ shares,

C owns $\frac{15\times36}{41}=\frac{540}{41}=13\frac{7}{41}$ shares,

D owns $\frac{10\times36}{41}=\frac{360}{41}=8\frac{32}{41}$ shares,

E owns $\frac{1\times36}{41}=\frac{36}{41}=\frac{36}{41}$ share,

$$\text{F owns } \frac{3\times 36}{41} = \frac{108}{41} = 2\tfrac{26}{41} \text{ shares,}$$

$$\text{G owns } \frac{2\times 36}{41} = \frac{72}{41} = 1\tfrac{31}{41} \text{ shares.}$$

$$\text{Total} = 36 \text{ shares.}$$

NOTE.—The syndicate formulæ are elastic, both with respect to the original number composing the syndicate and by the number each particular syndicate is increased or diminished. By analogy with the given cases, a formula can be easily written to fit any case.

FORMULA No. I.—Geometrical Progression

Auxiliary to Formulæ Nos. IV and VII.

Let $a+am+am^2+ \ldots +am^{n-1}$ represent the sum of n terms in geometrical progression in which a represents the first term, n the number of terms, m the ratio, and s the sum of n terms, then,

$$s=a+am+am^2+ \ldots +am^{n-1}. \quad \ldots \ldots \quad (1)$$

Multiply by m,

$$ms=am+am^2+am^3+ \ldots +am^{n-1}+am^n. \quad \ldots \ldots \quad (2)$$

Subtract (1) from (2),

$$ms-s=am^n-a,$$

or

$$s(m-1)=a(m^n-1).$$

$$\therefore \quad s=\frac{a(m^n-1)}{m-1}. \quad \text{(No. I.)}$$

EXAMPLE

Find the sum of $3+6+12+24+48+96+192+384$.

Apply Formula No. I,

$$s=\frac{a(m^n-1)}{m-1}.$$

Here a (first term) $=3$, m (ratio) $=2$, n (No. of terms) $=8$.

$$s=\frac{3(2^8-1)}{2-1};$$

$$s=\frac{3(256-1)}{1};$$

$$s=3\times 255;$$

$$\therefore \quad s=765. \quad \text{Answer.}$$

NOTE.—The convenience of this formula is more readily appreciated where n is large.

FORMULA No. II.—For Computing Tables No. 7 to 15 incl.

To find the amount of $\$P$ at compound interest at $r\%$ for n years.

Let A represent the amount;

P represent the principal;

r represent the rate;

n represent the number of years,

then

$A = P + Pr = P(1+r)$ amount at end of 1st year.

$\therefore \quad A = P(1+r)^n$ amount at end of nth year.

When P = 1,

$$\mathbf{A = (1+r)^n}. \quad \text{No. II } (a).$$

If interest is compounded:

Semi-annually, $A = (1 + \frac{1}{2}r)^{2n}$. No. II (b).

Quarterly, $A = (1 + \frac{1}{4}r)^{4n}$. No. II (c).

Bi-monthly, $A=(1+\frac{1}{6}r)^{6n}$. No. II (*d*).

Monthly, $A=(1+\frac{1}{12}r)^{12n}$. No. II (*e*).

EXAMPLE

(Beyond the limits of Table No. 11.)

Find the amount of \$16,500 for 18 years at 4% interest, compounded quarterly.

Apply Formula No. II (*c*),

$$A=P(1+\tfrac{1}{4}r)^{4n}.$$

Here $P=16{,}500$, $\frac{1}{4}r=.01$, $4n=4\times18=72$.

$$\therefore\quad A=16{,}500(1.01)^{72} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

[To find $(1.01)^{72}$ use logs.] $72 \log 1.01=0.3111389136=\log 2.047099318$.

$$\therefore\quad (1.01)^{72}=2.047099318. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

Substituting (*b*) in (*a*),

$$A=16{,}500\ (2.047099318),$$

$$\therefore\quad A=\$33{,}777.14. \quad \text{Answer.}$$

FORMULA No. III.—For Computing Tables No. 16 to 24 incl.

To find the present value, P, of \$$A$ payable in n years at $r\%$, which must just amount in n years to A.

Let A represent the amount;
P represent the principal;
r represent the rate;
n represent the number of years.

From Formula No. II, $$P=\frac{A}{(1+r)^n},$$

when $A=1$, $$\mathbf{P=\frac{1}{(1+r)^n}}.$$ No. III (a).

If interest is compounded:

Semi-annually, $$P=\frac{1}{(1+\frac{1}{2}r)^{2n}}.$$ No. III (b).

Quarterly $$P=\frac{1}{(1+\frac{1}{4}r)^{4n}}.$$ No. III (c).

Bi-monthly $$P=\frac{1}{(1+\frac{1}{6}r)^{6n}}.$$ No. III (d).

Monthly $P=\frac{1}{(1+\frac{1}{12}r)^{12n}}$. No. III (*e*).

EXAMPLE

(Beyond the limits of Table No. 18.)

Find the principal that will amount to \$20,000 in 25 years at 3% interest, compounded semi-annually.

Apply Formula No. III (*b*),

$$P=\frac{A}{(1+\frac{1}{2}r)^{2n}}.$$

Here $A=20{,}000$, $\frac{1}{2}r=.015$, $2n=2\times25=50$.

$$\therefore\quad P=\frac{20{,}000}{(1.015)^{50}}.$$

[To find $(1.015)^{50}$ use logs.] $50 \log 1.015=0.3233021100=\log 2.105242409$.

$$\therefore\quad (1.015)^{50}=2.105242409.$$

$$\therefore\quad P=\frac{20{,}000}{2.105242409}=\$9500.10.\quad \text{Answer.}$$

FORMULA NO. IV.—FOR COMPUTING TABLES NO. 25 TO 33 INCL.

To find the amount of $\$P$ successively deposited at regular intervals for n years at $r\%$, interest compounded at each regular interval.

Let A represent the amount;
P represent the principal;
r represent the rate;
n represent the number of years,

then

$A=P+Pr=P(1+r)=$amount at end of 1st year. . . . . (1)

$A=[P+P(1+r)][1+r]=P[(1+r)+(1+r)^2]=$amount at end of 2d year. . (2)

$\therefore A=P[(1+r)+(1+r)^2+\ .\ .\ .\ +(1+r)^n]=$amount at end of nth year. . (3)

Let $s=[(1+r)+(1+r)^2+\ .\ .\ .\ +(1+r)^n]$ a geometrical progression. . . (4)

Apply Formula No. I,

$$s=\frac{a(m^n-1)}{m-1},$$

$$a=(1+r),\ m=(1+r),$$

$$n=n,\ m-1=r.$$

$$\therefore \quad s=\frac{[1+r][(1+r)^n-1]}{r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Substituting (5) in (3),

$$A=\frac{P[1+r][(1+r)^n-1]}{r}. \quad \text{No. IV } (a).$$

When $P=1$,

$$A=\frac{[1+r][(1+r)^n-1]}{r}. \quad \text{No. IV } (b).$$

NOTE.—See note following Formula No. V.

EXAMPLE

(Beyond the limits of Table No. 25.)

What will \$750 deposited every 6 months for 23 years amount to, interest compounded semi-annually at $2\frac{1}{2}\%$ per annum?

Apply Formula No. IV (*a*),

$$A=\frac{P[1+r][(1+r)^n-1]}{r}.$$

Here $P=750$, $\frac{1}{2}r=.0125$ ($\frac{1}{2}$ of $2\frac{1}{2}\%$), $2n=2\times 23=46$.

$$A=\frac{750[1.0125][(1.0125)^{46}-1]}{.0125}=750[81][(1.0125)^{46}-1].$$

[To find $(1.0125)^{46}$ use logs.] $46 \log 1.0125=0.2481714674=\log 1.770807968$.

$$\therefore \quad [(1.0125)^{46}-1]=.770807968.$$

$$A=(750)\times(81)\times(.770807968).$$

$$\therefore \quad A=\$46{,}826.58. \quad \text{Answer.}$$

FORMULA NO. V.—For Computing Tables No. 34 to 42 Incl.

To find the sum P, which, if successively deposited at regular intervals, must just amount in n years, at $r\%$ interest, compounded at each regular interval, to $\$A$.

From Formula No. IV (a),

$$A = \frac{P[1+r][(1+r)^n-1]}{r}.$$

$$\therefore \quad P = \frac{Ar}{[1+r][(1+r)^n-1]}. \quad \text{No. V } (a).$$

When $A=1$,

$$\mathbf{P} = \frac{\mathbf{r}}{[1+\mathbf{r}]\ (1+\mathbf{r})^n-1]}. \quad \text{No. V } (b).$$

Note.—In formulæ Nos. IV and V if interest is compounded and sum is deposited:

Semi-annually, r becomes $\frac{1}{2}r$ and n becomes $2n$.

Quarterly, r becomes $\frac{1}{4}r$ and n becomes $4n$;

Bi-monthly, r becomes $\frac{1}{6}r$ and n becomes $6n$;

Monthly, r becomes $\frac{1}{12}r$ and n becomes $12n$.

EXAMPLE

(Beyond the limits of Table No. 36.)

What sum must be deposited each month to amount to \$2,000,000 in 50 years, interest compounded monthly at 3% per annum?

Apply Formula No. V (*a*),

$$P=\frac{Ar}{[1+r][(1+r)^n-1]}.$$

Here $A=2{,}000{,}000$, $\frac{1}{12}r=.0025$ ($\frac{1}{12}$ of 3%), $2n=600$ (50 years = 600 months).

$$\therefore\quad P=\frac{2{,}000{,}000\times.0025}{(1.0025)([1.0025]^{600}-1)}=\frac{2{,}000{,}000}{401[(1.0025)^{600}-1]}.$$

[To find $(1.0025)^{600}$ use logs.] $600 \log 1.0025=0.6506287800=\log 4.473307777$.

$$\therefore\quad [(1.0025)^{600}-1]=3.473307777.$$

$$P=\frac{2{,}000{,}000}{401[3.473307777]}=\frac{2{,}000{,}000}{1392.796418577}.$$

$$\therefore\quad P=\$1435.97.\quad \text{Answer.}$$

FORMULA NO. VI.—For Computing Table No. 43

To find the time required at any given rate for any amount to equal any given multiple of the principal.

Let m represent the multiple, then

$$A = mP. \quad \ldots \ldots \ldots \ldots \quad (1)$$

From Formula No. II,

$$A = P(1+r)^n. \quad \ldots \ldots \ldots \ldots \quad (2)$$

Substitute in (2) mP for A, then

$$mP = P(1+r)^n.$$

Divide by P,

$$m = (1+r)^n,$$

$$n \log (1+r) = \log m.$$

$$\therefore \quad \mathbf{n = \frac{\log m}{\log (1+r)}}. \quad \textbf{No. VI.}$$

EXAMPLE

(Beyond the limits of Table No. 43.)

What time is required for a sum of money to treble itself at 6% interest, compounded annually?

Apply Formula No. VI,

$$n=\frac{\log m}{\log (1+r)}.$$

Here $m=3$, $r=.06$, then

$$n=\frac{\log 3}{\log 1.06}=\frac{0.4771212547}{0.0253058653}=18.8542.$$

$.8542\times 360=307.512$,

.8542 yr. = 307.512 days,

307.512 days = 10 mos., 8 days. $\therefore$ $n=18$ yrs., 10 mos. and 8 days. Answer.

$\therefore$ Any sum of money will treble itself in 18 yrs., 10 mos. and 8 days at 6% interest, compounded annually.

FORMULA NO. VII

An annuity is a sum of money to be paid at regular intervals of time, as years, half years, quarter years.

To find the present value of an annuity of $\$A$ per annum for n years, at $r\%$.

The present value of 1st payment is $\frac{A}{(1+r)}$.

The present value of 2d payment is $\frac{A}{(1+r)^2}$.

The present value of nth payment is $\frac{A}{(1+r)^n}$.

$\therefore$ The present value of all the payments is $\frac{A}{1+r}+\frac{A}{(1+r)^2}+\ \ldots\ +\frac{A}{(1+r)^n}$.

$\left[\text{By Formula No. I, } s=\frac{a(m^n-1)}{m-1}=\frac{a(1-m^n)}{1-m}\right],\ a=\frac{A}{1+r},\ m=\frac{1}{1+r},\ n=n.$

$$\therefore\ \frac{A}{1+r}+\frac{A}{(1+r)^2}+\ \ldots\ +\frac{A}{(1+r)^n}=\frac{\frac{A}{1+r}\left[1-\left(\frac{1}{1+r}\right)^n\right]}{1-\frac{1}{1+r}}=\frac{A}{r}\left[\frac{(1+r)^n-1}{(1+r)^n}\right].$$

EXAMPLE

ANNUITY. FORMULA No. VII.

Find the present value of an annuity of \$500 for 5 years, if money is worth 5%.

Apply Formula No. VII,

$$P=\frac{A}{r}\left[\frac{(1+r)^n-1}{(1+r)^n}\right].$$

Here $A=500$, $r=.05$, $n=5$.

$$\therefore \quad P=\frac{500}{.05}\left[\frac{(1.05)^5-1}{(1.05)^5}\right]=10{,}000\left[\frac{(1.05)^5-1}{(1.05)^5}\right].$$

[To find $(1.05)^5$ use logs.] $5 \log 1.05=0.1059464955=\log 1.276281563.$

$$\therefore \quad (1.05)^5=1.276281563.$$

$$P=\frac{(10{,}000)\times(.276281563)}{1.276281563}=\frac{2762.81563}{1.276281563}.$$

$$\therefore \quad P=\$2164.74. \quad \text{Answer.}$$

LOGARITHMS OF NUMBERS

Common System

The logarithm of a number is the exponent that denotes the power to which a given number must be raised to produce the number:—thus, $10^3 = 10 \times 10 \times 10 = 1000$. That is, 3 is the logarithm of 1000.

In the above example, 10 is the base, 1000 is the number and 3 is the exponent. The exponent denotes how many times the base is taken as a factor.

The functions of logarithms are manifold, but their practical application in this work will be confined to the method of obtaining the powers of numbers.

It may be said that multiplication is a short method of addition and that one of the uses of logarithms is a short cut to multiplication: e.g., if we wish to find by multiplication the 17th power of 1.035, written $(1.035)^{17}$, the result would contain 51 decimal places, be laborious in the extreme, with the possibility of errors large; whereas by logarithms we would only have to multiply the log of 1.035 by 17 and from the table find for the answer the number corresponding to this logarithm: thus

$$\log 1.035 = 0.0149403498;$$

$$17 \log 1.035 = 0.2539859466;$$

$$\log 1.794675552 = 0.2539859466.$$

$$\therefore \quad (1.035)^{17} = 1.794675552.$$

As $10^0 = 1$, therefore $\log 1 = 0$;

$10^1 = 10$, therefore $\log 10 = 1$;

$10^2 = 100$, therefore $\log 100 = 2$;

$10^3 = 1000$, therefore $\log 1000 = 3$;

$10^4 = 10000$, therefore $\log 10000 = 4$

and so forth.

From the above it will be seen that the logarithms of numbers
between 1 and 10 lie between 0 and 1;
between 10 and 100 lie between 1 and 2;
between 100 and 1000 lie between 2 and 3;
between 1000 and 10000 lie between 3 and 4;
and so forth.

e.g., $\log 472.39 = 2$ plus a decimal.

The whole number (2 in this case) is called the characteristic and the decimal is called the mantissa.

Characteristic

When the number is greater than one the characteristic is positive and is one less than the number of digits to the left of the decimal point. This is easily written by inspection; thus, log 1,000,000. = 6, as one million has 7 digits to the left of the decimal point.

Mantissa

The logarithm of all numbers not integral powers of 10 contains a decimal called the mantissa in addition to the characteristic. The mantissa only is obtained from the table of logarithms.

To Find the Logarithm of a Number

Rule.—Write the characteristic. Take from the table the logarithm corresponding to the first five significant digits of the number. Considering the following digits as a decimal, multiply by the tabular difference, and adding this product to the logarithm taken will give the required logarithm.

To Find the Number Corresponding to a Given Logarithm

Rule.—Find in the table the next less mantissa, the five figures of the number corresponding thereto and the tabular difference. Subtract this mantissa from the given mantissa. Divide the difference just found by the tabular difference and annex the quotient to the first five figures, omitting the decimal point obtained in dividing. Place the decimal point so that the number of integral digits will be one greater than the characteristic.

Explanation of the Table

Mantissæ only of logarithms are given in the table. In the first column the first four digits of consecutive numbers are given, the fifth digit appearing in one of ten columns headed (0 to 9 inclusive) under which and horizontally from the first four digits of the numbers, appear their respective logarithms. To the right of this table is found the table of differences arranged in ten columns from 0 to 9 inclusive. This table of differences is given to save the trouble of subtracting one logarithm from the next following, to obtain the tabular difference and is brought

into use when interpolation becomes necessary. Where the number contains only five digits or less the necessity for interpolating in finding its logarithm does not arise, but in finding the logarithm of a number containing six or more digits, or in finding the number corresponding to a given logarithm it is necessary to have recourse to the principle of interpolation.

Example in Both Cases

Find the 16th power of $2.8945 = (2.8945)^{16}$.

The characteristic is one less than one $= 0$.

16 log 2.8945 $= 16 \times$ (0. plus a decimal).

In the table find number 2894, then opposite this number and under column headed 5 will be found the logarithm required, which is .4615735539; prefixing the characteristic gives 0.4615735539.

Multiplying by 16

27694413234

4615735539

$\therefore$ log of $(2.8945)^{16} = 7.3851768624$

The mantissa is .3851768624

Next less mantissa in table .3851592386 = log 24275

(**Interpolation**) Tabular difference 178902)176238.00000(98511

1610118

1522620

1431216

914040

894510

195300

178902

163980

178902

Annexing 98511 to 24275 and placing the decimal point so as to have eight integral digits, as the characteristic is 7, we have 24275985.11 as the 16th power of 2.8945.

Find 102d power of $1.0058\frac{1}{3}$, written $(1.0058\frac{1}{3})^{102}$.

Log of $1.0058 = 0.0025116313$

(Interpolation) $\frac{1}{3}$ Tab. Dif. $= \frac{1}{3}$ of $431768 = 143922\frac{2}{3}$

$\therefore$ log of $1.0058\frac{1}{3} = 0.0025260235\frac{2}{3}$

Multiply $\times 102$

102

68

50520470

252602350

$\therefore$ log $(1.0058\frac{1}{3})^{102} = 0.2576544038$

next less mantissa in table $.2576305839 = \log 18098$

(Interpolation) Tab. dif. 239961) 238199.00000(99265

2159649

2223410

2159649

637610

479922

1576880

1439766

1371140

1199805

annexing 99265 to 18098 and placing the decimal point so as to have one digit to the left, as the characteristic is 0, we have $1.809899265 = (1.0058\frac{1}{3})^{102}$.

The appearance of this treatise on logarithms is purely incidental and purposely a partial one. Without their aid the solution of the various formulæ containing the high powers of numbers would be difficult and, as has been shown, laborious and impracticable.

The example on page 131 and the two examples on pages 138, 139 and 140, show the extent of their application in this work and demonstrate, where similar computations are required, their superiority over all other known methods.

Recognized mathematicians are the authority for the statement that no table under ten places is sufficiently accurate for finding the high powers of numbers.

The logarithmic table used in this work was compiled by the United States Coast and Geodetic Survey and appears in their Report for 1896, Part II, Appendix No. 12.

In conclusion will say that I am indebted to Dr. Hodgkins and Prof. Peck of George Washington University for valuable suggestions and to employees of the Library of Congress for many courtesies.

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